The following problems cover the skills that are necessary to be successful on Test A.

1. Simplify: $\sqrt[3]{\dfrac{-16x^3}{2y^6}}$.

$$\sqrt[3]{\frac{-16x^3}{2y^6}} = \left(\frac{-16x^3}{2y^6}\right)^{1/3} = \left(\frac{-16}{2}\right)^{1/3} \cdot \left(x^3\right)^{1/3} \cdot \left(\frac{1}{y^6}\right)^{1/3}$$

$$= (-8)^{1/3} \cdot x^{3/3} \cdot \left(\frac{1^{1/3}}{y^{6/3}}\right)$$

$$= -2 \cdot x \cdot \frac{1}{y^2} = \boxed{\frac{-2x}{y^2}}$$

2. Perform the indicated operations and simplify: $\left(m^{n+1}r^n\right)\left(3m^n r^{2n}\right)^{-1}$.

$$\left(m^{n+1}r^n\right)\left(3m^n r^{2n}\right)^{-1} = m^{n+1}r^n \cdot \frac{1}{3m^n r^{2n}} = \frac{m^{n+1}r^n}{3m^n r^{2n}} = \frac{m^n \cdot m^1 \cdot r^n}{3m^n \cdot r^{n+n}} = \frac{m^n}{3m^n} \cdot m \cdot \frac{r^n}{r^n \cdot r^n}$$

$$= \frac{1}{3} \cdot m \cdot \frac{1}{r^n} = \boxed{\frac{m}{3r^n}}$$

3. Perform the indicated operations and simplify: $\dfrac{ab}{\frac{1}{a}+\frac{1}{b}}$.

$$\frac{ab}{\frac{1}{a}+\frac{1}{b}} = \frac{ab}{\left(\frac{b}{b}\right)\frac{1}{a} + \frac{1}{b}\left(\frac{a}{a}\right)}\left(\frac{a}{a}\right)\left(\frac{b}{b}\right) = \frac{\frac{a^2b^2}{ab}}{\frac{b}{ab} + \frac{a}{ab}} = \frac{\frac{a^2b^2}{ab}}{\frac{b+a}{ab}} = \frac{a^2b^2}{\cancel{ab}} \cdot \frac{\cancel{ab}}{b+a}$$

$$= \frac{a^2b^2}{b+a} = \boxed{\frac{a^2b^2}{a+b}}$$

4. Rationalize the denominator: $\dfrac{2}{\sqrt{2}+b}$.

$$= \frac{2}{\sqrt{2}+b} \cdot \frac{\sqrt{2}-b}{\sqrt{2}-b} = \boxed{\frac{2\sqrt{2}-2b}{2-b^2}}$$

5. Evaluate $(5x+1)^{3/4} - (7-x)^0$ for x = 3.

$$(5x+1)^{3/4} - (7-x)^0$$

$$= (5(3)+1)^{3/4} - (7-(3))^0$$

$$= (15+1)^{3/4} - (4)^0$$

$$= (16)^{3/4} - 1$$

$$= 8 - 1 = \boxed{7}$$

6. Evaluate $-\left(2b^2\right)^{-1}$ when $b=-2$.

$$-\left(2b^2\right)^{-1} = -\left(2(-2)^2\right)^{-1} = -\left(2(4)\right)^{-1} = \boxed{-\frac{1}{8}}$$

7. Simplify completely: $2\sqrt{50}-7\sqrt{18}+\sqrt{8}$.

$$2\sqrt{50} - 7\sqrt{18} + \sqrt{8}$$
$$= 2\sqrt{25\cdot 2} - 7\sqrt{9\cdot 2} + \sqrt{4\cdot 2}$$
$$= 2\cdot 5\sqrt{2} - 7\cdot 3\sqrt{2} + 2\sqrt{2}$$
$$= 10\sqrt{2} - 21\sqrt{2} + 2\sqrt{2}$$
$$= \boxed{-9\sqrt{2}}$$

8. Simplify completely: $2u\left(3u^2-1\right)-\left(-8u^3-14u+6\right)$.

$$2u(3u^2-1) - (-8u^3-14u+6)$$
$$= 2u(3u^2) - 2u(1) + 8u^3 + 14u - 6$$
$$= 6u^3 - 2u + 8u^3 + 14u - 6$$
$$= 6u^3 + 8u^3 - 2u + 14u - 6$$
$$= \boxed{14u^3 + 12u - 6}$$

9. Simplify completely: $4(2x+1)^2+3(2x+1)+1$.

$$4(2x+1)^2 + 3(2x+1) + 1$$
$$= 4(2x+1)(2x+1) + 3(2x) + 3(1) + 1$$
$$= 4(4x^2 + 2x + 2x + 1) + 6x + 3 + 1$$
$$= 4(4x^2 + 4x + 1) + 6x + 4$$
$$= 16x^2 + 16x + 4 + 6x + 4$$
$$= \boxed{16x^2 + 22x + 8}$$

10. Factor completely: $32x^4y-162y$.

$$32x^4y - 162y$$
$$= 2y(16x^4 - 81)$$
$$= 2y(4x^2+9)(4x^2-9)$$
$$= \boxed{2y(4x^2+9)(2x+3)(2x-3)}$$

11. Perform the indicated operation and simplify completely:

$$\frac{z^2+z-12}{2z^2+6z} * \frac{z^2+3z}{6z+24}$$

$$\frac{z^2+z-12}{2z^2+6z} = \frac{(z-3)(z+4)}{2z(z+3)}$$

$$\frac{z^2+3z}{6z+24} = \frac{z(z+3)}{6(z+4)}$$

$$\frac{(z-3)\cancel{(z+4)}}{2z\cancel{(z+3)}} \cdot \frac{z\cancel{(z+3)}}{6\cancel{(z+4)}} = \frac{z-3}{2z} \cdot \frac{z}{6} = \frac{\cancel{z}(z-3)}{12\cancel{z}}$$

$$= \boxed{\frac{z-3}{12}}$$

12. Perform the indicated operation and simplify: $\dfrac{3c}{c-2}+\dfrac{c+1}{2-c}$.

$$\frac{3c}{c-2}+\frac{c+1}{2-c}=\frac{3c}{c-2}+\frac{c+1}{-(c-2)}=\frac{-3c}{-(c-2)}+\frac{c+1}{-(c-2)}$$
$$=\frac{-3c+c+1}{-(c-2)}=\frac{-2c+1}{-(c-2)}=\boxed{\frac{2c-1}{c-2}}$$

13. Solve for z: 7z-(4z-9)=24+5(z-1)

$$7z-(4z-9)=24+5(z-1)$$
$$=7z-4z+9=24+5z-5$$
$$=3z+9=19+5z$$
$$=-10=2z$$
$$\boxed{-5=z}$$

14. Solve for x:

$$\frac{a}{3}+5x=b\left(\frac{x}{3}+2\right)$$

$$3\cdot\left(\frac{a}{3}+5x=\frac{bx}{3}+2b\right)\cdot 3$$
$$=a+15x=bx+6b$$
$$=15x-bx=6b-a$$
$$=x(15-b)=6b-a$$
$$=\boxed{x=\frac{6b-a}{15-b}}$$

15. Solve for t: $2t^2+4t=9t+18$.

$$2t^2+4t=9t+18$$
$$=2t^2+4t-9t-18=0$$
$$=2t^2-5t-18=0$$
$$=(2t-9)(t+2)=0$$

$2t-9=0$ $\quad$ $t+2=0$

$2t=9$ $\quad$ $t=-2$ $\quad$ $\boxed{t=\frac{9}{2},-2}$

$t=\frac{9}{2}$

16. Solve for s: $-2s^2-4s+2s^3=0$.

$$-2s^2-4s+2s^3=0$$
$$=2s(-s-2+s^2)=0$$
$$=2s(s^2-s-2)=0$$
$$=2s(s-2)(s+1)=0$$

$2s=0$ $\quad$ $s-2=0$ $\quad$ $s+1=0$

$s=0$ $\quad$ $s=2$ $\quad$ $s=-1$

$$\boxed{s=0,2,-1}$$

17. Solve for p: $\frac{4}{p} - \frac{2}{p+1} = 3$.

$p\left(\frac{4}{p} - \frac{2}{p+1}\right) = 3(p)$

$= 4 - \frac{2p}{p+1}(p+1) = 3p(p+1)$

$= 4(p+1) - 2p = 3p(p+1)$

$= 4p+4-2p = 3p^2+3p$

$= 2p+4 = 3p^2+3p$

$3p^2+p-4=0$

$(3p+4)(p-1)=0$

$p=-\frac{4}{3}$ $\quad$ $p=1$ $\quad$ $\boxed{p=-\frac{4}{3}, 1}$

18. To get a B in a course a student must have an average of at least 80% on five tests that are worth 100 points each. On the first four tests a student scores 92%, 83%, 61%, and 71%. Determine the lowest score the student can receive on the fifth test to assure a grade of B for the course.

B = 80% average on 5 tests; each test is worth 100 points

let x represent the lowest score on the 5th test to receive a B in the class.

$$(5)\ 0.80 = \frac{\overset{\text{test 1}}{0.92} + \overset{\text{test 2}}{0.83} + \overset{\text{test 3}}{0.61} + \overset{\text{test 4}}{0.71} + \overset{\text{test 5}}{x}}{5}\ (5)$$

$4.00 = 3.07 + x$

$0.93 = x$

The student must get at least a $\boxed{93\%}$ on the fifth test to receive a B in the class.

19. The area of a rectangle is 84 square feet and the length is 6 feet longer than the width. If w represents the width, write an equation that could be used to find the dimensions of the rectangle.

$A = l \cdot w = 84\text{ ft}^2$

$l = w+6$

$84 = (w+6)\cdot w$

$\boxed{w(w+6)=84}$

20. A furniture store drops the price of a table 37 percent to a sale price of $364.77. What is the original price?

Let P represent the original price.

$P - 0.37P = 364.77$

$P(1-0.37) = 364.77$

$\frac{P(0.63)}{0.63} = \frac{364.77}{0.63} \longrightarrow \boxed{P=579}$

21. Solve for t: $(t+2)^2 = 8$.

$\sqrt{(t+2)^2} = \sqrt{8}$

$t+2 = \pm\sqrt{8}$

$t = -2 \pm \sqrt{8}$

$t = -2 \pm \sqrt{4\cdot 2}$

$\boxed{t = -2 \pm 2\sqrt{2}}$

22. Solve for z: $z^2 - 4z + 6 = 0$.

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a=1 \quad b=-4 \quad c=6$$

$$z = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(6)}}{2(1)}$$

$$z = \frac{4 \pm \sqrt{16-24}}{2} = \frac{4 \pm \sqrt{-8}}{2} = \frac{4 \pm \sqrt{4 \cdot (-2)}}{2} = \frac{4 \pm 2\sqrt{-2}}{2} = 2 \pm \sqrt{-2} \rightarrow \boxed{z = 2 \pm i\sqrt{2}}$$

23. Perform the indicated operation and simplify: $\sqrt{-2} \cdot \sqrt{-24}$.

$$\sqrt{-2} \cdot \sqrt{-24} = \sqrt{(-2)(-24)} = \sqrt{48} = \sqrt{16 \cdot 3} = \boxed{4\sqrt{3}}$$

24. Solve for r: $5 - 3r \le 8$.

$$5 - 3r \le 8$$

$$\frac{-3r}{-3} \le \frac{3}{-3}$$

$$\boxed{r \ge -1}$$

25. Solve for x: $|2x+1| \ge 7$.

$$|2x+1| \ge 7$$

$-(2x+1) \ge 7$ or $(2x+1) \ge 7$

$-2x - 1 \ge 7$ $\qquad$ $2x \ge 6$

$-2x \ge 8$ $\qquad$ $x \ge 3$

$x \le -4$

$$\boxed{(-\infty, -4] \quad \text{or} \quad [3, \infty)}$$

26. Find the domain of $y = \sqrt{4 - 5x}$.

$$0 \le \sqrt{4-5x}$$

$$0 \le 4 - 5x$$

$$5x \le 4$$

$$x \le \frac{4}{5} \rightarrow \boxed{(-\infty, \tfrac{4}{5}]}$$

27. Find the x-intercepts of $y - 2x^2 - 13x = 6$.

$$y - 2x^2 - 13x = 6$$

$$0 - 2x^2 - 13x = 6$$

$$2x^2 + 13x + 6 = 0$$

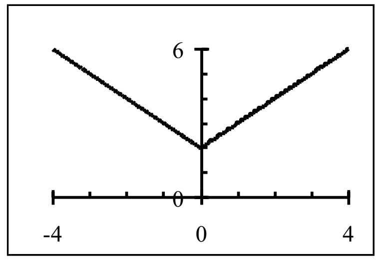
$$(2x + 1)(x + 6) = 0$$

$x = -\frac{1}{2}$ $\qquad$ $x = -6$

$(-\frac{1}{2}, 0)$ $\qquad$ $(-6, 0)$

$$\boxed{(-\tfrac{1}{2}, 0), (-6, 0)}$$

28. Find the equation of the graph at the right:



$y = mx + b$

m is slope
b is y-intercept

we can see that it intersects the y-axis at $y=2$ so $b=2$.

$m = \frac{rise}{run} = \frac{4}{4} = 1$ and $\frac{4}{-4} = -1$

because of the vertex on the y-axis, we can see that we are taking the absolute value of x, so the equation is: $\boxed{y = |x| + 2}$

29. Find the distance between (6,3) and (-2,4).

$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(-2-6)^2 + (4-3)^2} = \sqrt{(-8)^2 + (1)^2} = \sqrt{64+1} = \boxed{\sqrt{65}}$

30. Find the midpoint of the line segment joining (6,9) and (-3,1).

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{6-3}{2}, \frac{9+1}{2}\right) = \left(\frac{3}{2}, \frac{10}{2}\right) = \boxed{\left(\frac{3}{2}, 5\right)}$

31. Find the slope and y-intercept of the line $5x + 4y = 8$.

$5x + 4y = 8 \rightarrow 4y = -5x + 8 \rightarrow y = -\frac{5}{4}x + 2$

$y = mx + b$ where m is the slope and b is the y-intercept, so the slope is $\boxed{m = \frac{5}{4}}$ and the y-intercept is $\boxed{b = 2.}$

32. Find the equation of the line perpendicular to $3y + 2x - 3 = 0$ passing through (4,-1).

$3y + 2x - 3 = 0$
$3y = -2x + 3$
$y = -\frac{2}{3}x + 1$

perpendicular line slope $= \frac{3}{2}$

$y - (-1) = \frac{3}{2}(x - 4)$
$y = \frac{3}{2}x - 6 - 1 \rightarrow y = \frac{3}{2}x - 7$
$2y = 3x - 14$
$\boxed{2y - 3x + 14 = 0}$

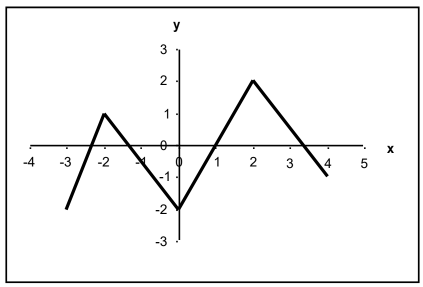
33. Find $f(-4)$ if $f(x) = \dfrac{2x^2 - 11}{3x}$

$$f(x) = \frac{2x^2-1)}{3x} \longrightarrow f(-4) = \frac{2(-4)^2 - 11}{3(-4)} = \frac{2(16)-11}{-12} = \frac{32-11}{-12} = \frac{21}{-12} \rightarrow \boxed{f(-4) = \frac{-7}{4}}$$

34. Find $f(b+2)$ if $f(x) = 5 - 3(x+1)$.

$$f(x) = 5 - 3(x+1)$$
$$f(b+2) = 5 - 3((b+2)+1)$$
$$f(b+2) = 5 - 3(b+3)$$
$$f(b+2) = 5 - 3b - 9$$
$$\boxed{f(b+2) = -3b - 4}$$

35. Find the domain and the range of the function graphed at the right:



The domain is the inputs, or x-values, and the range is the outputs, or y-values.

domain : [-3, 4]
range : [-2, 2]

36. If (5,6) is a point on the graph of $y = g(x)$, find a point on the inverse graph, $g^{-1}(x)$

$$y = g(x) \rightarrow 6 = g(5) \rightarrow 5 = g^{-1}(6)$$

$$\boxed{(6, 5)}$$

37. If $h(t) = \dfrac{t}{t+1}$, find the value of t so that $h(t) = 3$.

$$h(t) = \frac{t}{t+1} \; ; \; h(t) = 3 \rightarrow 3 = \frac{t}{t+1}$$

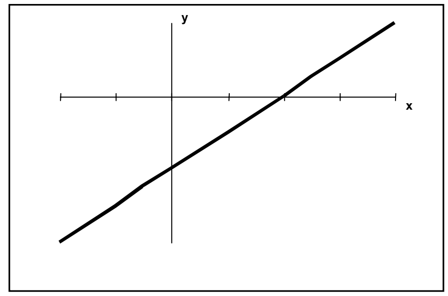
$$(t+1)\,3 = \frac{t}{t+1}(t+1)$$

$$= \quad 3(t+1) = t$$

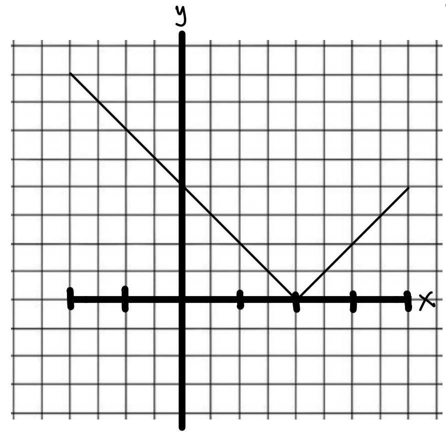
$$= 3t + 3 = t$$
$$= \quad 3 = -2t$$

$$\boxed{-\frac{3}{2} = t}$$

38. If the graph of $y = f(x)$ is at the right, sketch the graph of $y = |f(x)|$.



when we take the absolute value of a negative number, it becomes positive, so wherever the y-value is negative on the original graph, it is positive on the new graph.



39. Rewrite $10^b = a$ in logarithmic form.

$$10^b = a$$
$$\log_{10}(10^b) = \log_{10}(a)$$
$$b = \log_{10} a$$
$$\boxed{\log a = b}$$

40. Rewrite as a single logarithm: $\frac{1}{2}\log x + 4\log y - 2\log z$.

$$\frac{1}{2}\log x + 4\log y - 2\log z$$
$$= \log(x^{1/2}) + \log y^4 - \log(z^2) \quad \text{(power rule)}$$
$$= \log(\sqrt{x}) + \log(y^4) + \log(\tfrac{1}{z^2}) \quad \text{(quotient rule)}$$
$$= \log(\sqrt{x} \cdot y^4 \cdot \tfrac{1}{z^2}) = \boxed{\log\left(\frac{\sqrt{x}\,y^4}{z^2}\right)} \quad \text{(product rule)}$$

41. Solve for t: $3^{2t} = 27^{2t-1}$.

$$3^{2t} = 27^{2t-1}$$
$$3^{2t} = (3^3)^{2t-1}$$
$$3^{2t} = 3^{3(2t-1)}$$
$$2t = 3(2t-1)$$
$$2t = 6t - 3$$
$$+3 \qquad +3$$
$$2t + 3 = 6t$$
$$-2t \qquad -2t$$
$$3 = 4t$$

$$\frac{3}{4} = \frac{4t}{4}$$
$$\boxed{\frac{3}{4} = t}$$

42. Solve the system of equations:

$$\begin{cases} 4x + 3y = 0 \\ 8x = 9y + 2 \end{cases}$$

$4x + 3y = 0 \longrightarrow (4x + 3y = 0) \cdot 3 \rightarrow 12x + 9y = 0$

$8x = 9y + 2 \longrightarrow 8x - 9y = 2 \longrightarrow + (8x - 9y = 2)$

$\frac{20x}{20} = \frac{2}{20}$

$x = \frac{2}{20}$

$\boxed{x = \frac{1}{10}}$

$(4x + 3y = 0) \cdot -2$

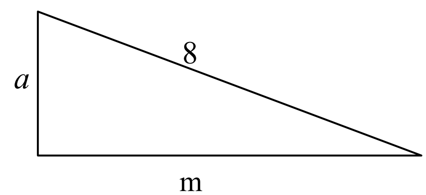
$8x - 9y = 2$

$+ (-8x - 6y = 0)$

$\frac{-15y}{-15} = \frac{2}{-15}$

$\boxed{y = -\frac{2}{15}}$

43. Express the length of side *a* in terms of *m*:



$a^2 + m^2 = 8^2$

$a^2 + m^2 = 64$

$a^2 = 64 - m^2$

$\boxed{a = \sqrt{64 - m^2}}$