Some Precalculus Problems

1. Express the area of a circle, *A*, in terms of its circumference, *C*.

$$A = \pi r^2$$
$$C = 2\pi r \rightarrow \frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$
$$\frac{C}{2\pi} = r$$
$$A = \pi r^2 = \left(\frac{C}{2\pi}\right)^2 = \pi\left(\frac{C^2}{2^2\pi^2}\right) = \frac{C^2\pi}{4\pi^2} = \frac{C^2}{4\pi} \rightarrow \boxed{A = \frac{C^2}{4\pi}}$$

2. Simplify: $\sqrt[3]{\frac{-16x^3}{2y^6}}$.

$$\sqrt[3]{\frac{-16x^3}{2y^6}} = \left(\frac{-16x^3}{2y^6}\right)^{1/3} = \left(\frac{-16}{2}\right)^{1/3}\cdot(x^3)^{1/3}\cdot\left(\frac{1}{y^6}\right)^{1/3}$$
$$= (-8)^{1/3}\cdot x^{3/3}\cdot\left(\frac{1}{y^{6/3}}\right)$$
$$= -2\cdot x\cdot\frac{1}{y^2} = \boxed{\frac{-2x}{y^2}}$$

3. Perform the indicated operations and simplify: $\left(m^{n+1}r^n\right)\left(3m^nr^{2n}\right)^{-1}$.

$$(m^{n+1}r^n)(3m^nr^{2n})^{-1} = m^{n+1}r^n\cdot\frac{1}{3m^nr^{2n}} = \frac{m^{n+1}r^n}{3m^nr^{2n}} = \frac{m^n\cdot m^1\cdot r^n}{3m^n\cdot r^{n+n}} = \frac{m^n}{3m^n}\cdot m\cdot\frac{r^n}{r^n\cdot r^n}$$
$$= \frac{1}{3}\cdot m\cdot\frac{1}{r^n} = \boxed{\frac{m}{3r^n}}$$

4. Perform the indicated operations and simplify: $\frac{ab}{\frac{1}{a}+\frac{1}{b}}$.

$$\frac{ab}{\frac{1}{a}+\frac{1}{b}} = \frac{ab}{\left(\frac{b}{b}\right)\frac{1}{a}+\frac{1}{b}\left(\frac{a}{a}\right)}\left(\frac{a}{a}\right)\left(\frac{b}{b}\right) = \frac{\frac{a^2b^2}{ab}}{\frac{b}{ab}+\frac{a}{ab}} = \frac{\frac{a^2b^2}{ab}}{\frac{b+a}{ab}} = \frac{a^2b^2}{ab}\cdot\frac{ab}{b+a}$$
$$= \frac{a^2b^2}{b+a} = \boxed{\frac{a^2b^2}{a+b}}$$

5. Find $f^{-1}(x)$ for $f(x) = \frac{1-3x}{4}$.

$$f(x) = \frac{1-3x}{4}$$
$$y = \frac{1-3x}{4} \quad \text{solve for } x.$$
$$(4)\,y = \left(\frac{1-3x}{4}\right)\cdot 4$$
$$4y = 1-3x$$
$$4y = 1-3x$$
$$+3x \qquad +3x$$
$$3x+4y = 1$$
$$-4y \qquad -4y$$
$$\frac{3x}{3} = \frac{1-4y}{3}$$
$$x = \frac{1-4y}{3} \rightarrow \boxed{f^{-1}(x) = \frac{1-4x}{3}}$$

6. Evaluate $(5x+1)^{3/4}-(7-x)^0$ for $x=3$.

$(5x+1)^{3/4} - (7-x)^0$

$= (5(3)+1)^{3/4} - (7-(3))^0$

$= (15+1)^{3/4} - (4)^0$

$= (16)^{3/4} - 1$

$= 8 - 1 = \boxed{7}$

7. Evaluate $-\left(2b^2\right)^{-1}$ when $b=-2$.

$-(2b^2)^{-1} = -(2(-2)^2)^{-1} = -(2(4))^{-1} = \boxed{-\frac{1}{8}}$

8. Find the interval where $g(x) > 0$ if $g(x) = -x^2 - x + 6$.

$g(x) = -x^2 - x + 6$

$(-x+2)(x+3)$

$-x+2>0$ $\quad$ $x+3>0$

$2>x$ $\quad$ $x>-3$

$x<2$

$\boxed{(-3, 2)}$

9. If $f(t)=\dfrac{2}{1-t}$, for what value of t does $f(t)=3$?

$f(t) = \dfrac{2}{1-t}$

$(1-t)3 = \left(\dfrac{2}{1-t}\right)\cdot 1-t$

$3(1-t) = 2$

$3-3t = 2$

$3-3t = 2$

$-3 \quad -3$

$-3t = -1$

$\boxed{t = \frac{1}{3}}$

10. Simplify completely: $2u\left(3u^2-1\right)-\left(-8u^3-14u+6\right)$.

$2u(3u^2-1)-(-8u^3-14u+6)$

$= 2u(3u^2) - 2u(1) + 8u^3 + 14u - 6$

$= 6u^3 - 2u + 8u^3 + 14u - 6$

$= 6u^3 + 8u^3 - 2u + 14u - 6$

$= \boxed{14u^3 + 12u - 6}$

11. Simplify completely: $4(2x+1)^2+3(2x+1)+1$.

$4(2x+1)^2 + 3(2x+1)+1$

$= 4(2x+1)(2x+1) + 3(2x) + 3(1) + 1$

$= 4(4x^2+2x+2x+1) + 6x + 3 + 1$

$= 4(4x^2+4x+1) + 6x + 4$

$= 16x^2 + 16x + 4 + 6x + 4$

$= \boxed{16x^2 + 22x + 8}$

12. Factor completely: $32x^4y - 162y$.

$$32x^4y - 162y$$
$$= 2y(16x^4 - 81)$$
$$= 2y(4x^2+9)(4x^2-9) = \boxed{2y(4x^2+9)(2x+3)(2x-3)}$$

13. What is the remainder when $5x^2 - 2x + 1$ is divided by $x - 1$?

$$\begin{array}{r} 5x + 3 \\ x-1 \overline{)\,5x^2 - 2x + 1} \\ -(5x^2 - 5x) \\ \hline 3x + 1 \\ -(3x - 3) \\ \hline \boxed{4} \end{array}$$

14. Find a so that the two lines do not intersect: $y = 4x + 2$, $y - 3 = ax$.

$y = 4x + 2 \parallel y - 3 = ax$

$\parallel$ lines have the same slope

$y - 3 = ax$

$\Rightarrow y = ax + 3$

slope of $y = 4x + 2$ is 4 so slope of $y = ax + 3$ must also be 4. $\therefore$ $\boxed{a = 4}$

15. Perform the indicated operation and simplify: $\dfrac{4m^2 - v^2}{3m - 1} \div \dfrac{2m^2 + mv}{3m - 1}$.

$$\frac{4m^2 - v^2}{3m-1} = \frac{(2m-v)(2m+v)}{3m-1}$$

$$\frac{2m^2 + mv}{3m-1} = \frac{m(2m+v)}{3m-1}$$

$$\frac{4m^2 - v^2}{3m-1} \div \frac{2m^2+mv}{3m-1} = \frac{(2m-v)\cancel{(2m+v)}}{\cancel{3m-1}} \times \frac{\cancel{3m-1}}{m\cancel{(2m+v)}}$$
$$= \boxed{\frac{2m-v}{m}}$$

16. Perform the indicated operation and simplify: $\dfrac{3c}{c-2} + \dfrac{c+1}{2-c}$

$$\frac{3c}{c-2} + \frac{c+1}{2-c} = \frac{3c}{c-2} + \frac{c+1}{-(c-2)} = \frac{-3c}{-(c-2)} + \frac{c+1}{-(c-2)}$$
$$= \frac{-3c + c + 1}{-(c-2)} = \frac{-2c+1}{-(c-2)} = \boxed{\frac{2c-1}{c-2}}$$

17. Simplify completely:

$$\frac{\frac{a}{x}-\frac{x}{a}}{\frac{1}{a}-\frac{1}{x}} = \left(\frac{a}{a}\right)\frac{\frac{a}{x}-\frac{x}{a}}{\frac{1}{a}-\frac{1}{x}\left(\frac{a}{a}\right)} = \frac{\frac{a^2}{ax}-\frac{x}{a}\left(\frac{x}{x}\right)}{\left(\frac{x}{x}\right)\frac{1}{a}-\frac{a}{ax}} = \frac{\frac{a^2}{ax}-\frac{x^2}{ax}}{\frac{x}{ax}-\frac{a}{ax}} = \frac{\frac{a^2-x^2}{ax}}{\frac{x-a}{ax}}$$

$$\frac{\frac{a^2-x^2}{ax}}{\frac{x-a}{ax}} = \frac{a^2-x^2}{ax}\cdot\frac{ax}{x-a} = \frac{(a+x)(a-x)}{x-a} = \frac{(a+x)(a-x)}{-(a-x)}$$

$$= -(a+x)$$

$$= \boxed{-a-x}$$

18. Solve for z: $7z-(4z-9)=24+5(z-1)$.

$$7z-(4z-9)=24+5(z-1)$$
$$=7z-4z+9=24+5z-5$$
$$=3z+9=19+5z$$
$$=-10=2z$$
$$\boxed{-5=z}$$

19. Solve for x: $\frac{a}{3}+5x=b(\frac{x}{3}+2)$

$$3\cdot\left(\frac{a}{3}+5x=\frac{bx}{3}+2b\right)\cdot 3$$
$$=a+15x=bx+6b$$
$$=15x-bx=6b-a$$
$$=x(15-b)=6b-a$$
$$=\boxed{x=\frac{6b-a}{15-b}}$$

20. Solve for r: $S=\frac{2r-a}{r-1}$.

$$(r-1)\ S=\frac{2r-a}{r-1}\ (r-1)$$
$$S(r-1)=2r-a$$
$$Sr-S=2r-a$$
$$+S \qquad +S$$
$$Sr=2r+S-a$$
$$-2r \qquad -2r$$
$$Sr-2r=S-a$$
$$\frac{r(S-2)}{S-2}=\frac{S-a}{S-2}$$
$$\boxed{r=\frac{S-a}{S-2}}$$

21. Solve for R: $V = \frac{3R}{a} - \frac{R}{b}$

$$V = \frac{3R}{a}\left(\frac{b}{b}\right) - \frac{R}{b}\left(\frac{a}{a}\right)$$

$$V = \frac{3Rb}{ab} - \frac{Ra}{ab} = \frac{3Rb - Ra}{ab} \cdot ab$$

$$(ab)V = 3Rb - Ra$$

$$\frac{abV}{3b-a} = \frac{R(3b-a)}{3b-a}$$

$$\boxed{\frac{abV}{3b-a} = R}$$

22. Solve for t: $2t^2 + 4t = 9t + 18$.

$$2t^2 + 4t = 9t + 18$$

$$= 2t^2 + 4t - 9t - 18 = 0$$

$$= 2t^2 - 5t - 18 = 0$$

$$= (2t-9)(t+2) = 0$$

$2t - 9 = 0$ $\quad$ $t + 2 = 0$

$2t = 9$ $\quad$ $t = -2$

$t = \frac{9}{2}$

$$\boxed{t = \frac{9}{2}, -2}$$

23. Solve for s: $-2s^2 - 4s + 2s^3 = 0$.

$$-2s^2 - 4s + 2s^3 = 0$$

$$= 2s(-s - 2 + s^2) = 0$$

$$= 2s(s^2 - s - 2) = 0$$

$$= 2s(s-2)(s+1) = 0$$

$2s = 0$ $\quad$ $s - 2 = 0$ $\quad$ $s + 1 = 0$

$s = 0$ $\quad$ $s = 2$ $\quad$ $s = -1$

$$\boxed{s = 0, 2, -1}$$

24. Solve for m: $m^3 + 3m^2 - 4m - 12 = 0$.

$$m^3 + 3m^2 - 4m - 12 = 0$$
$$m^2(m+3) - 4(m+3) = 0$$
$$(m^2-4)(m+3) = 0$$
$$(m+2)(m-2)(m+3) = 0$$

$m+2=0$ $m-2=0$ $m+3=0$

$m=-2$ $m=2$ $m=-3$

$\boxed{m = 2, -2, -3}$

25. Solve for p:

$$\frac{4}{p} - \frac{2}{p+1} = 3$$

$$p\left(\frac{4}{p} - \frac{2}{p+1}\right) = 3(p)$$
$$= 4 - \frac{2p}{p+1} (p+1) = 3p\,(p+1)$$
$$= 4(p+1) - 2p = 3p(p+1)$$
$$= 4p+4-2p = 3p^2+3p$$
$$= 2p+4 = 3p^2+3p$$
$$3p^2 + p - 4 = 0$$
$$(3p+4)(p-1) = 0$$

$p = -\frac{4}{3}$ $p = 1$

$\boxed{p = -\frac{4}{3}, 1}$

26. To get a B in a course a student must have an average of at least 80% on five tests that are worth 100 points each. On the first four tests a student scores 92%, 83%, 61%, and 71%. Determine the lowest score the student can receive on the fifth test to assure a grade of B for the course.

B = 80% average on 5 tests; each test is worth 100 points

let x represent the lowest score on the 5th test to receive a B in the class.

test 1, test 2, test 3, test 4, test 5

$$(5)\ 0.80 = \frac{0.92 + 0.83 + 0.61 + 0.71 + x}{5}\ (5)$$
$$4.00 = 3.07 + x$$
$$0.93 = x$$

The student must get at least a $\boxed{93\%}$ on the fifth test to receive a B in the class.

27. The area of a rectangle is 84 square feet and the length is 6 feet longer than the width. If w represents the width, write an equation that could be used to find the dimensions of the rectangle.

$$A = l \cdot w = 84\,ft^2$$
$$l = w + 6$$
$$84 = (w+6) \cdot w$$

$\boxed{w(w+6) = 84}$

28. A furniture store drops the price of a table 37 percent to a sale price of \$364.77. What is the original price?

Let P represent the original price.

$P - 0.37P = 364.77$

$P(1-0.37) = 364.77$

$\frac{P(0.63)}{0.63} = \frac{364.77}{0.63}$ → $\boxed{P = 579}$

29. The cost of mailing envelopes by bulk mail is \$35 for the first 200 plus \$0.12 for each additional envelope over 200. Write a function to represent the cost of mailing x envelopes when $x \geq 200$.

first 200 = \$35

201 = \$35 + 0.12(1) = \$35 + 0.12(201 − 200)

202 = \$35 + 0.12(2) = \$35 + 0.12(202 − 200)

cost function = $\boxed{C(x) = \$35 + 0.12(x-200)}$

30. Solve for t: $(t+2)^2 = 8$.

$\sqrt{(t+2)^2} = \sqrt{8}$

$t+2 = \pm\sqrt{8}$

$t = -2 \pm \sqrt{8}$

$t = -2 \pm \sqrt{4 \cdot 2}$

$\boxed{t = -2 \pm 2\sqrt{2}}$

31. Solve for y: $-15y + 6y^2 = -y$.

$-15y + 6y^2 = -y$

$+y \qquad\qquad +y$

$6y^2 - 14y = 0$

$y(6y-14) = 0$

$y = 0 \qquad 6y - 14 = 0$

$\frac{6y}{6} = \frac{14}{6}$

$y = \frac{14}{6} = \boxed{\frac{7}{3}}$

32. Solve for z: $z^2 - 4z + 6 = 0$.

$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 1 \quad b = -4 \quad c = 6$

$z = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(6)}}{2(1)}$

$z = \frac{4 \pm \sqrt{16-24}}{2} = \frac{4 \pm \sqrt{-8}}{2} = \frac{4 \pm \sqrt{4 \cdot (-2)}}{2} = \frac{4 \pm 2\sqrt{-2}}{2} = 2 \pm \sqrt{-2}$ → $\boxed{z = 2 \pm i\sqrt{2}}$

33. If a solution to $f(x)=0$ is $x=5$, find a solution to $3f(x+2)=0$.

$f(5)=0$ ← → $\frac{3f(x+2)}{3}=\frac{0}{3}$

$f(x+2)=0$

$f(5)=0 \quad f(x+2)=0$

$5=x+2$

$\boxed{3=x}$

34. Solve for x: $\sqrt{x+6}=x$

$(\sqrt{x+6})^2=(x)^2$

$x+6=x^2$

$-x-6 \quad -x-6$

$0=x^2-x-6$

$0=(x-3)(x+2)$

$x-3=0 \qquad x+2=0$

$x=3 \qquad x=-2$

$x=3 \rightarrow \sqrt{3+6}=3 \qquad x=-2 \rightarrow \sqrt{-2+6}=-2$

$\sqrt{9}=3 \qquad \sqrt{4}=-2$

$3=3 \qquad 2\neq-2$

$\boxed{x=3}$ is our only solution

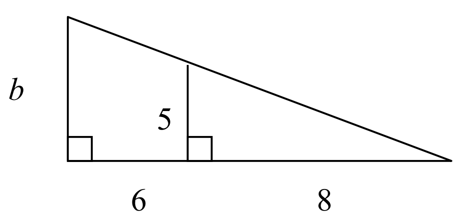
35. Solve for r: $5-3r\le 8$.

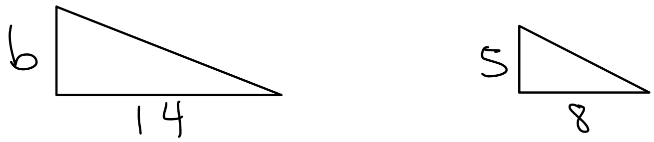
$5-3r\le 8$

$\frac{-3r}{-3}\le\frac{3}{-3}$

$\boxed{r\ge -1}$

36. Find the length of b:





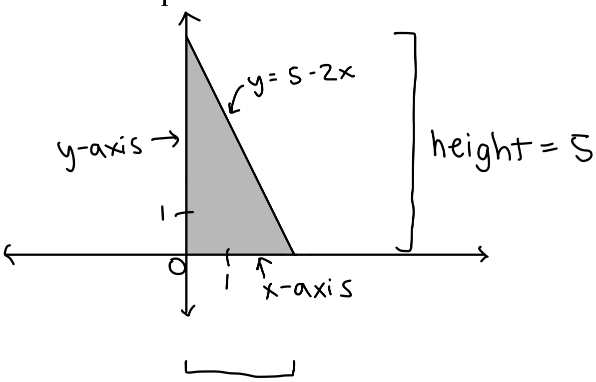
$(6)\frac{5}{6}=\frac{8}{14}(6)$

$(14)5=\frac{8b}{14}(14)$

$\frac{70}{8}=\frac{8b}{8}$

$\boxed{\frac{35}{4}}=\frac{70}{8}=b$

37.Find the area of the triangle bounded by $y = 5 - 2x$, the x-axis, and the y-axis in the first quadrant.



Area of a triangle $= \frac{1}{2}(\text{base})(\text{height})$
$= \frac{1}{2}\left(\frac{5}{2}\right)(5)$
$= \frac{1}{2}\left(\frac{25}{2}\right)$
$= \frac{25}{4}$

base : $y = 5 - 2x$ meets the x-axis
$0 = 5 - 2x$
$2x = 5$
$x = \frac{5}{2}$

38. Solve for x: $|2x+1| \geq 7$.

$|2x+1| \geq 7$

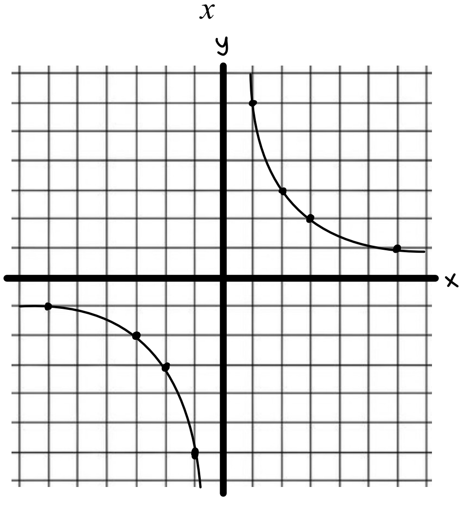
$-(2x+1) \geq 7$ or $(2x+1) \geq 7$
$-2x - 1 \geq 7$ $\quad$ $2x \geq 6$
$-2x \geq 8$ $\quad$ $x \geq 3$
$x \leq -4$

$(-\infty, -4]$ or $[3, \infty)$

39. Find the domain of $y = \sqrt{4 - 5x}$.

$0 \leq \sqrt{4-5x}$
$0 \leq 4 - 5x$
$5x \leq 4$
$x \leq \frac{4}{5}$ $\rightarrow$ $(-\infty, \frac{4}{5}]$

40. Graph $y = \dfrac{6}{x}$.



when $x = 1$, $y = 6$
when $x = 2$, $y = 3$
when $x = 3$, $y = 2$
when $x = 6$, $y = 1$

when $x = -1$, $y = -6$
when $x = -2$, $y = -3$
when $x = -3$, $y = -2$
when $x = -6$, $y = -1$

41. Find the intercepts of $y - 2x^2 - 13x = 6$.

x-intercepts where $y = 0$

$0 - 2x^2 - 13x = 6$

$2x^2 + 13x + 6 = 0$

$(2x + 1)(x + 6) = 0$

$x = -\frac{1}{2}$ $\quad$ $x = -6$

$\boxed{(-\frac{1}{2}, 0)}$ $\quad$ $\boxed{(-6, 0)}$

y-intercept where $x = 0$

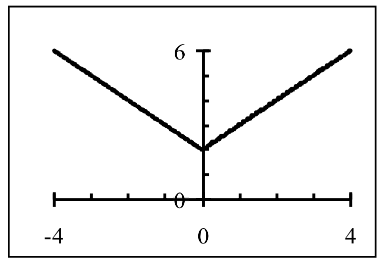
$y - 2(0)^2 - 13(0) = 6$

$y - 0 - 0 = 6$

$y = 6$

$\boxed{(0, 6)}$

42. Find the equation of the graph :



$y = mx + b$ $\quad$ m is slope

b is y-intercept

we can see that it intersects the y-axis at $y = 2$ so $b = 2$.

$m = \frac{\text{rise}}{\text{run}} = \frac{4}{4} = 1$ and $\frac{4}{-4} = -1$

because of the vertex on the y-axis, we can see that we are taking the absolute value of x, so the equation is: $\boxed{y = |x| + 2}$

43. Find the distance between (6,3) and (-2,4).

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-2-6)^2 + (4-3)^2} = \sqrt{(-8)^2 + (1)^2} = \sqrt{64 + 1} = \boxed{\sqrt{65}}$

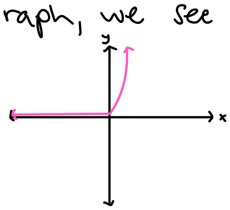
44. Find the midpoint of the line segment joining (6,9) and (-3,1).

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{6-3}{2}, \frac{9+1}{2}\right) = \left(\frac{3}{2}, \frac{10}{2}\right) = \boxed{\left(\frac{3}{2}, 5\right)}$

45. What is the range of $y = 2(3)^t$?

When looking at a graph, we see that there is an asymptote at $y = 0$.

The graph looks like :



we can conclude that the range is $\boxed{y > 0.}$

46. Find the equation of the line perpendicular to $3y + 2x - 3 = 0$ passing through (4,-1).

$3y + 2x - 3 = 0$

$3y = -2x + 3$

$y = -\frac{2}{3}x + 1$

perpendicular line slope $= \frac{3}{2}$

$y - (-1) = \frac{3}{2}(x - 4)$

$y = \frac{3}{2}x - 6 - 1 \rightarrow y = \frac{3}{2}x - 7$

$2y = 3x - 14$

$\boxed{2y - 3x + 14 = 0}$

47. Find $f(-4)$ if $f(x)=\dfrac{2x^2-11}{3x}$.

$f(x)=\dfrac{2x^2-1)}{3x} \longrightarrow f(-4)=\dfrac{2(-4)^2-11}{3(-4)} = \dfrac{2(16)-11}{-12} = \dfrac{32-11}{-12} = \dfrac{21}{-12} \rightarrow \boxed{f(-4)=\dfrac{-7}{4}}$

48. Find $f(b+2)$ if $f(x)=5-3(x+1)$.

$f(x)=5-3(x+1)$

$f(b+2)=5-3((b+2)+1)$

$f(b+2)=5-3(b+3)$

$f(b+2)=5-3b-9 \longrightarrow \boxed{f(b+2)=-3b-4}$

49. Find the domain of $g(x)=\dfrac{1}{x^2-x-12}$.

$x^2-x-12\neq 0$

$(x-4)(x+3)\neq 0$

$x-4\neq 0 \qquad x+3\neq 0$

$x\neq 4 \qquad x\neq -3$

$\boxed{\{x \mid x\neq 4,-3\}}$

50. Find $h(3)$ if $h(t)\begin{cases}2t^2-5 & t<-1\\ 4-3t & t\geq -1\end{cases}$

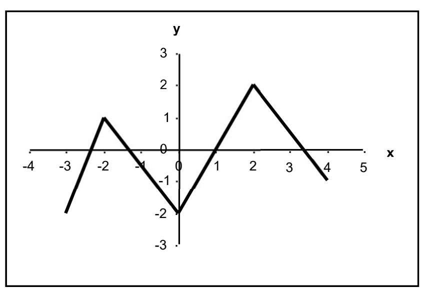
$3 = t \geq -1$

$h(3)=4-3(3)$

$h(3)=4-9$

$\boxed{h(3)=-5}$

51. Find the domain and the range of the function:



The domain is the inputs, or x-values, and the range is the outputs, or y-values.

domain: [-3, 4]
range: [-2, 2]

52. If (5,6) is a point on the graph of $y=g(x)$, find a point on the graph of $y=-g(x)+1$.

$6=g(5)$

$y=-g(5)+1$

$y=-(6)+1$

$y=-5$

$\boxed{(5,-5)}$

53. Find $g(f(-2))$ if $f(x) = \log_4(-8x)$ and $g(x) = x - 3$.

$f(-2) = \log_4(-8(-2))$
$= \log_4(16) \rightarrow x = \log_4(16)$
$4^x = 16$
$x = 2$
$= 2$

$g(2) = 2 - 3 = -1$

$g(f(-2)) = -1$

54. If $h(t) = \dfrac{t}{t+1}$, find the value of t so that $h(t) = 3$.

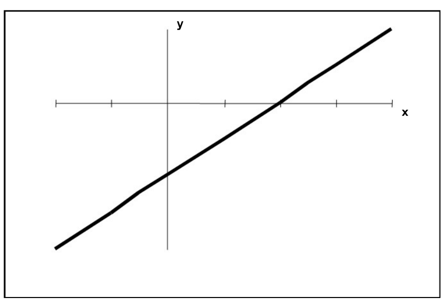
$h(t) = \frac{t}{t+1}$; $h(t) = 3 \rightarrow 3 = \frac{t}{t+1}$

$(t+1)\,3 = \frac{t}{t+1}(t+1)$

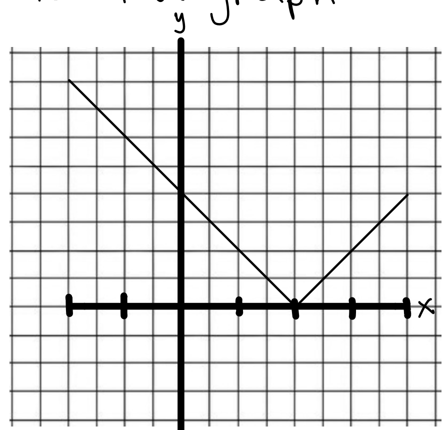
$= 3(t+1) = t$
$= 3t + 3 = t$
$= 3 = -2t$

$-\frac{3}{2} = t$

55. If the graph of $y = f(x)$ is below, sketch the graph of $y = |f(x)|$.

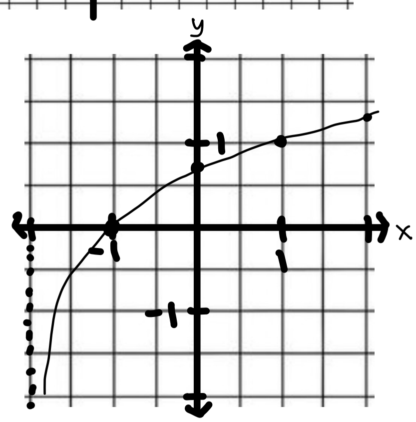


when we take the absolute value of a negative number, it becomes positive, so wherever the y-value is negative on the original graph, it is positive on the new graph.



56. Sketch the graph of $y = \log_3(x + 2)$.

when $x = -2$, $y = \log_3(0)$, undefined
when $x = -1$, $y = \log_3(1)$, $y = 0$
when $x = 0$, $y = \log_3(2) = \frac{\log(2)}{\log(3)}$, $y \approx 0.63$
when $x = 1$, $y = \log_3(3)$, $y = 1$
when $x = 2$, $y = \log_3(4) = \frac{\log(4)}{\log(3)}$, $y \approx 1.26$



57. Rewrite $5^b = a$ in logarithmic form.

$5^b = a$
$\log_5(5^b) = \log_5(a)$

$b = \log_5(a)$

58. Rewrite as a single logarithm: $\frac{1}{2}\log x + 4\log y - 2\log z$.

$$\frac{1}{2}\log x + 4\log y - 2\log z$$

$= \log(x^{1/2}) + \log y^4 - \log(z^2)$ (power rule)

$= \log(\sqrt{x}) + \log(y^4) + \log(\frac{1}{z^2})$ (quotient rule)

$= \log(\sqrt{x} \cdot y^4 \cdot \frac{1}{z^2}) = \boxed{\log\left(\frac{\sqrt{x}\,y^4}{z^2}\right)}$ (product rule)

59. Solve for t: $3^{2t} = 27^{2t-1}$.

$3^{2t} = 27^{2t-1}$

$3^{2t} = (3^3)^{2t-1}$

$3^{2t} = 3^{3(2t-1)}$

$2t = 3(2t-1)$

$2t = 6t - 3$

$+3 \quad +3$

$2t + 3 = 6t$ →

$2t + 3 = 6t$

$-2t \quad -2t$

$\frac{3}{4} = \frac{4t}{4}$

$\boxed{\frac{3}{4} = t}$

60. Solve for r: $3 + 6e^{2r} = 5$.

$3 + 6e^{2r} = 5$

$-3 \quad -3$

$\frac{6e^{2r}}{6} = \frac{2}{6}$

$e^{2r} = \frac{2}{6}$

$\ln(e^{2r}) = \ln(\frac{2}{6})$

$2r = \ln(\frac{1}{3})$

$r = \frac{\ln(\frac{1}{3})}{2}$ → $\boxed{r = \frac{-\ln(3)}{2}}$

61. Solve for y: $\log_3 y - \log_3(y-1) = 2$.

$\log_3 y - \log_3(y-1) = 2$

$= \log_3 y + \log_3(\frac{1}{y-1}) = 2$ logarithm power identity

$= \log_3\left(\frac{y}{y-1}\right) = 2$ logarithm product identity

$= \left(\frac{y}{y-1}\right) = 3^2$

$= (y-1)\frac{y}{y-1} = 9\,(y-1)$

$= y = 9(y-1)$

$= y = 9y - 9$

$-9y \quad -9y$

$= -8y = -9$ →

$\frac{-8y}{-8} = \frac{-9}{-8}$

$\boxed{y = \frac{9}{8}}$

62. Solve the system of equations: $\begin{cases} 4x + 3y = 0 \\ 8x = 9y + 2 \end{cases}$

$4x + 3y = 0 \longrightarrow (4x + 3y = 0)\cdot 3 \rightarrow 12x + 9y = 0$

$8x = 9y + 2 \longrightarrow 8x - 9y = 2 \longrightarrow + (8x - 9y = 2)$

$\frac{20x}{20} = \frac{2}{20}$

$x = \frac{2}{20}$

$\boxed{x = \frac{1}{10}}$

$(4x + 3y = 0)\cdot -2$

$8x - 9y = 2$

$+ (-8x - 6y = 0)$

$\frac{-15y}{-15} = \frac{2}{-15}$

$\boxed{y = -\frac{2}{15}}$

63. If $f(x) = -x^2$ and $g(x) = x + 4$, find the values of x so that $g(f(x)) > 0$.

$g(f(x)) > 0$

$g(x) > 0$

$x + 4 > 0$

$-4 \quad -4$

$x > -4$

$f(x) > -4$

$\frac{-x^2}{-1} > \frac{-4}{-1}$

$\sqrt{x^2} < \sqrt{4}$

$\pm x < 2$

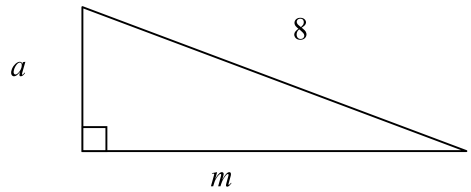
$x < 2$

$\frac{-x}{-1} < \frac{2}{-1}$

$x > -2$

$\boxed{-2 < x < 2}$

64. Express the length of side a in terms of m:



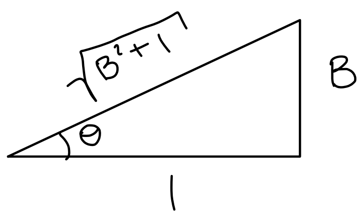
$a^2 + m^2 = 8^2$

$a^2 + m^2 = 64$

$a^2 = 64 - m^2$

$\boxed{a = \sqrt{64 - m^2}}$

65. If $\tan\theta = B$ where θ is an angle in quadrant I, express $\sin\theta$ in terms of B.

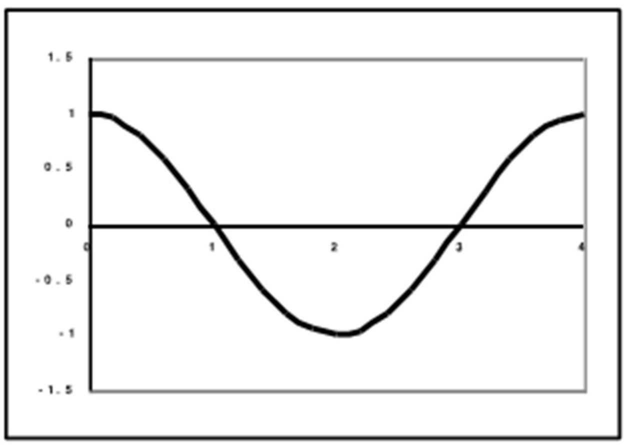


$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = B = \frac{B}{1}$

using pythagorean theorem, solve for the hypotenuse.

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \boxed{\frac{B}{\sqrt{B^2+1}}}$

66. Find the trigonometric equation for this graph:



This is a cosine curve because when $x=0$, $y=1$.

The amplitude is the vertical distance from the equilibrium line to the maximum of the curve. In this graph, the amplitude is from the x-axis to $y=1$, so the amplitude is 1.

The period is the length of one cycle. In this graph, the period is 4. $4 = \frac{2\pi}{B}$. $B = \frac{\pi}{2}$. $y = (\text{amplitude})\cos(Bx)$

$y = 1\cos\left(\frac{\pi}{2}x\right) \rightarrow \boxed{y = \cos\left(\frac{\pi x}{2}\right)}$

67. $\sin(\theta + \pi) =$

use the symmetry identity of $\sin(\theta+\pi)$, $\sin(\theta+\pi) = \boxed{-\sin(\theta)}$

68. Find $\cos\left(\frac{4\pi}{3}\right)$

$\cos\left(\frac{4\pi}{3}\right) = \cos(\pi) + \cos\left(\frac{\pi}{3}\right) = -1 + \frac{1}{2} = \boxed{-\frac{1}{2}}$