

Nonparametric Statistics on Manifolds- By Examples and Applications

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Abstract. The general theory of nonparametric statistics on manifolds M presented here is of recent origin. It builds much of its framework on the notion of the Fréchet mean of a probability measure Q , namely, the point on the manifold which minimizes the expected squared distance from a random variable with distribution Q . The nonparametric methods are intrinsic or extrinsic, depending on the distance used on M . The extrinsic distance is the distance induced from a good embedding of M in a Euclidean space, while the intrinsic distance is the geodesic distance on the manifold when endowed with a Riemannian structure. In examples, it is often the case that the nonparametric methods yield sharper inference than their parametric counterparts provide. Although we consider an application to paleomagnetism where M is the sphere S^2 , our main emphasis is on landmarks based shape spaces. The latter include (i) spaces of 2D and 3D images invariant under an appropriate group of transformations, which are useful in morphometrics and medical diagnostics, (ii) affine shape spaces invariant under affine transformations, useful in scene recognition based on satellite images, and (iii) projective shape spaces used in machine vision and robotics. We also briefly consider 2D continuous images, and nonparametric estimation of shape densities.