**Math 577 - Topics in Applied Mathematics - Fall 2004**

**Fourier Analysis and Approximation Theory**

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It is a standard result that there exist continuous functions $f$ whose Fourier series do not converge uniformly to $f$. After establishing this fact, we will analyse what additional minimal properties – besides continuity – the function $f$ must fulfill in order for uniform convergence to take place. These minimal properties are derived from approximation theory results.

This is a typical topic covered in this course: the interplay between Fourier analysis and approximation theory.

My plan for the course consists of covering the following topics.

1. The Dini-Lipschitz theorem and uniform convergence of Fourier Series.
2. Modulus of continuity – the classical Jackson approximation theorem. The converse theorems of Bernstein.
3. The Hilbert transform and convergence of Fourier series in $L_P$, $1 < P < \infty$. The conjugate Fourier series. The Calderon-Zygmund decomposition of $L_1$ functions.
4. Harmonic measure and the Riesz-Thorin interpolation theorem. The Hahn-Banach theorem. The Hausdorff-Young theorem for the Fourier transform and for the convolution.
5. The Bernstein theorem for Fourier series of analytic functions. The Paley-Wiener theorem. Application to spectral methods for PDE's.
6. Conformal mapping and the inequalities of Bernstein and Markoff – characterization of functions whose Fourier series converge at exponential speed.
7. The Abel and Cesaro methods of summability – Wiener's theory of saturation.
8. The Littlewood-Paley theory. Links with wavelets.

This course should be accessible to graduate students in mathematics, applied mathematics as well as those in various science and engineering disciplines.

More precisely exposure to basic real and complex analysis, at the level of Math 424 - 524 and Math 425 – 525, is adequate as prerequisites. Math 527 - Principles of Analysis - is a good substitute for the above courses. All the relevant results of functional analysis, Fourier analysis and approximation theory will be fully developed in due time. Hence prior knowledge of these topics, although useful, is not needed.

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