

THE ATIYAH-SINGER INDEX THEOREM

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The classical Atiyah-Singer Index Theorem provides a topological formula, in terms of characteristic classes, of the Fredholm index of certain elliptic operators. It bridges in an impressive way geometry and topology with functional analysis, and had influenced, since its discovery in the 1960's, many areas of mathematics. I plan to follow somehow the outline of the original Annals papers [1] and [2], but with more emphasis on Dirac and differential operators, rather than pseudo-differential operators. In the final part of the course I shall indicate some modern applications and approaches.

The course should be accessible and of interest to students that plan to work in geometry or analysis. The prerequisites are: one graduate level course in (differential) geometry and, ideally, one in functional analysis, but I shall cover everything that is needed from functional analysis.

Topics will include:

- (1) A motivation: signature and Hodge theory
- (2) Elliptic operators and Fredholm theory
- (3) Topological K -theory
- (4) Characteristic classes
- (5) The analytical index
- (6) The topological index
- (7) Proof of the theorem
- (8) C^* -algebras and their K -theory
- (9) Dirac operators and Spin structures
- (10) The tangent groupoid
- (11) A proof using C^* -algebras and the tangent groupoid
- (12) Other applications (time permitting)

REFERENCES

- [1] M. F. Atiyah and I. M. Singer, *The index of elliptic operators. I*, Ann. of Math. (2) **87** (1968), 484–530
- [2] M. F. Atiyah and I. M. Singer, *The index of elliptic operators. III*, Ann. of Math. (2) **87** (1968), 546–604.
- [3] N. Higson and J. Roe, *Analytic K -homology*, Oxford Science Publications, 2000.
- [4] N. Higson and J. Roe, *The Atiyah-Singer index theorem*, manuscript, 2005.
- [5] J. Roe, *Elliptic operators, topology and asymptotic methods*, 2nd edition, Longman, 1998.