

Title: Commutative Algebra

Instructor: Ana-Maria Castravet

Text: Commutative Algebra by M. Atiyah and I.G. MacDonald

(Commutative Algebra with a view towards algebraic geometry,
by D. Eisenbud, also recommended)

Required course: Algebra

Contents: Rings and Ideals, Modules, Localization,
Primary Decomposition, Integral Dependence, Chain Conditions, Completions,
Dimension Theory

Description: Commutative algebra is the foundation of modern
Algebraic Geometry and this course should be viewed as a preparatory
course for an Algebraic Geometry course. We plan to cover basic
topics in commutative algebra, while uncovering the geometry behind the various
notions and results.

We will start with rings and prime spectra of rings, modules, localization, associated
primes, the primary decomposition
theorem and its geometric interpretation, integral dependence and the Nullstellensatz.
We continue with noetherian rings and modules, completions and Hensel's Lemma.
After this we will introduce different characterizations of dimension, regular local
rings, discrete valuation rings, the Noether normalization theorem and its applications
to the dimension of affine rings. If time permits, we will discuss Grothendieck's generic
freeness lemma with its applications to the semicontinuity of fiber dimensions.