Course Proposal: Representations of p-adic Groups

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I propose to teach a graduate topics course in the Spring 2008 semester, called "Representations of p-adic Groups". This could be listed either as Math 518 Topics in Algebra or Math 519 Topics in Number Theory. The prerequisites will be the first-year graduate mathematics core courses. It would also be helpful to have seen some representation theory of finite groups, but the main ideas of this theory will be reviewed, and superseded by the material covered. The course should be of interest to any student who plans on studying number theory, arithmetic algebraic geometry, or any kind of representation theory. I have already spoken to at least 8 graduate students who would sign up for the course if it were offered.

There will be no one text, but various resources which I will lecture from, and which I plan to have put on reserve in the library for the semester.

The course will begin with several lectures on Representation Theory from a general viewpoint, and will be loosely based on the two articles by A. Knapp, "Group Representations and Harmonic Analysis from Euler to Langlands, Parts I and II", in the Notices, April and May 1996. The purpose of this introduction is to expose how broad of a subject Representation Theory is, and how it is used in many parts of mathematics. These lectures will conclude with a informal discussion of the Langlands conjectures, which are at the forefront of present research in number theory, and combine ideas from analysis, algebra, and geometry. The groups of Lie type over p-adic fields, or p-adic groups, play an integral role in these conjectures, and this will motivate the main topic of the course. The subject is interesting in its own right, as well, as its development involves tools from topics ranging from algebraic geometry and topology to finite groups of Lie type.

After the first motivating lectures, the material will begin from first principles, with a study of topological groups. This will conclude with a proof of the existence and uniqueness of the Haar measure on locally compact groups, a result which is not often covered in any course, but which is used in many aspects of harmonic analysis, as well as representation theory. The next several lectures will be on p-adic rings and fields, covering several points of view of their construction and significance, including both topological and number-theoretic viewpoints. After these basic topics are covered, the representation theory will begin, with a quick review of representations of finite groups. This will lead into the basic theory of the representations of compact groups (which included finite groups), following Part I of the book "Lie Groups" by Daniel Bump.

Following approximately 4 or 5 lectures on compact groups (with linear groups over p-adic rings as examples), we will begin the representation theory for linear groups over p-adic fields, or more generally, representations for locally compact totally disconnected groups. We will follow two main sources: Chapter 4 of "Automorphic Forms and Representations" by Daniel Bump, and the survey article "Representations of the group $GL(n, F)$ where F is a non-archimedean local field", in Russian Mathematical Surveys (Vol. 31, 1976), by I. N. Bernstein and A. V. Zelevinsky. The concentration will be on how the main concepts from representation theory of compact or finite groups, including Schur's Lemma, Frobenius Reciprocity, and Mackey Theory, may be extended to locally compact disconnected groups. We will go as far as the semester permits.