Integral Lattices, Linear Codes, and Finite Groups

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The theory of Euclidean integral lattices (a.k.a. positive definite integer-valued quadratic forms) is an important branch of number theory with deep connections to finite groups and representation theory, error-correcting codes and data transmission, combinatorics, geometry, chemistry and physics. In particular, the problem of finding dense lattices and more generally, dense sphere packings was mentioned by David Hilbert in 1900 in his list of open problems, and it is still open until now.

The main theme of the course is to utilize tools such as representation theory of finite groups and modular forms to derive a number of results, some classical and some very recent, about integral lattices.

This topic course should be of interest for the students specialized in algebra, number theory, and combinatorics. Prerequisites include first-year graduate algebra course and very basic knowledge about functions in complex variables.

Topics to be included:

1. Root lattices and lattice constructions using linear codes
2. Grothendieck group of integral lattices and Hasse-Minkowski theorem
3. Minkowski-Siegel mass formula and theorems of Conway-Thompson and Steinberg
4. Elkies theorem on long shadow lattices
5. Classification of unimodular lattices of rank up to 24 (due to Mordell-Witt, Kneser, Niemeier-Venkov)
6. Conway-Sloane theorem on odd unimodular lattices of rank 32
7. Cohn-Elkies-Kumar theorem on the optimality of the Leech lattice
8. Lempken-Schroeder-Tiep lower bound for the minimum of an integral lattice, and Bachoc-Tiep lower bound for the minum weight of a linear code
9. Elkies-Shioda constructions of dense lattices using elliptic curves and Gross' concept of globally irreducible lattices
10. Weil representations of finite classical groups
11. Tiep-Zalesskii theorem on unramified representations
12. Symplectic spreads and Scharlau-Tiep construction of symplectic group lattices.