Algebraic curves and their moduli

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Course Description: The main goal of this course is the study of algebraic curves and their variation in families.

One of the recurring ideas in algebraic geometry is to study properties of algebraic varieties by allowing them to vary in families and study 'parameter spaces' of classes of algebraic varieties (often themselves algebraic varieties; a typical example is the Grassmanian of k-dimensional subspaces in a given vector space). A moduli space is roughly speaking a parameter space whose points are in one-to-one correspondence with isomorphism classes of the members of a class of varieties. A moduli space encodes information about all families of varieties in the given class,
and often understanding general facts about families reduces to prove some specific facts about the moduli space and its geometry.

This course proposes to be an introduction to the moduli theory of algebraic curves, namely the Deligne-Mumford moduli space $\bar M_g$ of stable genus g curves. The moduli space of curves has proven itself a central object in geometry and its study touches on many major questions and areas in algebraic geometry. This course has as a secondary goal to be a gentle introduction to several of these areas, namely: general moduli theory, Hilbert schemes, deformation theory, Geometric Invariant Theory and aspects of birational geometry.

A concrete goal of the course will be to prove the classical result of Eisenbud-Harris-Mumford that $\bar M_g$ is a variety of general type for $g>22$ (An informal reformulation: 'it is not possible to write down an equation for a *general* curve of genus g>22'- in contrast with, for example, elliptic curves.) Another concrete goal of this course (if time permits) is to discuss Schafarevich's conjecture, in both its classical and number field versions, in connection to Faltings' proof of the Mordell conjecture.

Prerequisites: an Algebraic Geometry course.
Text: a good reference will be the book 'Moduli of Curves' by Joe Harris and Ian Morrison.