Proposal for a Topics Course
Iwasawa Theory

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This course will provide a basic introduction to an active area of research in algebraic number theory. One of the central concerns of algebraic number theory is the understanding of the structure of the class group of a number field. Iwasawa theory is concerned with how these groups grow as one climbs a tower of number fields. It turns out that this growth is quite regular, and can largely be described in terms of certain invariants. In turn, this growth is linked to analytic functions known as *L*-functions, and especially their p-adic counterparts. Moreover, Iwasawa theory extends to study the growth of other interesting objects, such as Selmer groups of elliptic curves.

Prerequisites for the course include graduate algebra and a basic course in graduate algebraic number theory (including class field theory). Some familiarity with elliptic curves is helpful, but not necessary. We will draw from several texts, as there is currently no complete text giving a comprehensive introduction to the subject:

L. Washington, Introduction to Cyclotomic Fields, 2nd Ed., Springer
R. Greenberg, Topics in Iwasawa Theory (book in preparation)
J. Neukirch, A. Schmidt, K. Wingberg, Cohomology of Number Fields, 2nd Ed., Springer
R. Greenberg, Introduction to Iwasawa theory for Elliptic Curves, in Arithmetic Algebraic Geometry (B. Conrad and K. Rubin, eds.), AMS

Topics to be covered, as time permits, include:

- Dirichlet characters, Bernoulli numbers, L-functions
- Kubota-Leopoldt p-adic L-functions
- Structure theory of Iwasawa modules
- Galois groups of (abelian) extensions unramified outside a set of primes: theorems and conjectures
- Topics in Galois cohomology: Kummer theory, duality theorems
- Main conjecture of Iwasawa theory (which is a theorem)
- Key ideas of two methods of proof of the main conjecture
- Brief introduction to Iwasawa theory of elliptic curves, Selmer groups

Bill McCallum and myself both do research in Iwasawa theory. Therefore, this course will benefit those wishing to learn more about the research of some of the faculty in the department.