INFINITE-DIMENSIONAL ANALYSIS

Spring 2011 topics course proposal

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Text: David Nualart, "Malliavin Calculus".

How does one do analysis in infinitely many dimensions? One needs: infinitely many variables, a rich class of functions which depend on them, a measure to integrate these functions and a way to differentiate them. A Wiener space is just the space. The variables are white noise variables, represented as the Wiener process---Brownian motion in physical terminology. The space carries the Wiener measure, which is the distribution of the process. Various interesting operations are defined on functions of Wiener paths, including stochastic integration and Malliavin derivative, which allows to differentiate Wiener functionals. These are all the tools we need.

If this sounds intriguing, take this course. I will develop systematically analysis of functionals on the Wiener space. Since its creation by Paul Malliavin in the seventies, the theory has found numerous applications to probability theory, analysis, mathematical physics and geometry. Remarkably, its mathematical formalism is very closely related to second quantization in field theory. I will start from the necessary probability and analysis background. Then I will introduce the main ideas and theorems of the Malliavin calculus. The rest of the course will be devoted to applications, notably to Malliavin's spectacular proof of Hoermander's hypoellipticity theorem---a major result in the theory of partial differential equations. Malliavin calculus is a beautiful, versatile and useful theory which shows how powerful probability and analysis are when used together.

What do you need in terms of preparation? Mainly enthusiasm to learn some exciting mathematics. Previous exposure to stochastic processes or quantum theory will help, but is not required. Talk to me if you are in doubt.

See you in the Spring!

Janek Wehr