Differential Geometry, General Relativity, and Cosmology

Special course on Math 488-588, Topics on Mathematical Physics, Spring 2011

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The course 488-588 will cover the following topics:

1. Tensor algebra
2. Special theory of relativity
3. Calculus of differential forms on manifolds.
4. Tensor analysis on manifolds. Affine connection and covariant differentiation.
5. Riemannian geometry, Curvature and Ricci tensors. Geodesics.
6. Riemannian spaces of diagonal curvature and their integrability. Spaces of constant curvature and flat connection. N-orthogonal coordinate systems.
7. Basic principles of general relativity. Einstein equations. The simplest solution of Einstein equations (Kasner metrics).
8. Gravitational waves.
9. Spherically-symmetric gravitational field. Black holes. Horizon. Volkov-Oppenheimer metrics.
10. Geodesics in the field of a black hole. Deflection of light in the gravitational field. Gravitational lenses.
11. Charged black holes. Naked singularity.
12. Rotating black hole: Kerr's solution. Black hole as a source of energy.
13. Gravitational solitons: the charged rotating black holes. Partial integrability of Einstein equations.
14. Are the travels in time possible?
15. Basic cosmological models. Red shift. Gravitational collapse.
16. Black matter and black energy. The predictable fate of the Universe.
17. Origin of the Universe. Inflation. Why our Universe is spatially flat?

The goal of this course is to give students the mathematical apparatus making it possible to deal with such urgent physical problems as basic cosmological models, the origin and predictable fate of Universe, and to explain why the concept of dark matter and dark energy is necessary to be in agreement with observable astronomic data. After taking this course, the student will get a solid knowledge of black holes, the resting ones as well as charged and rotating, and will be able to answer the question: why are travels in time not possible? More mathematically oriented students will probably concentrate on exact solutions of the Einstein equations and on other applications of the theory of integrable systems to differential geometry.

We will climb on these peaks of human intellectual achievements starting from pretty low and modest levels. First, we will study the tensor algebra in affine, Euclidean, and pseudo-Euclidean spaces, the calculus of differential forms on manifolds, and then the basic points of Riemannian geometry. Then we will spend most part of time on study of general relativity.

The course will be within reach for both graduate and undergraduate students from Mathematical, Physical, and Astronomy Departments. The prerequisites are modest: linear algebra (Math 415) and ordinary differential equations (Math 252). The knowledge in PDE is desirable but not necessary.

I will not follow strictly any particular textbook but will use from time to time three of them:

1. David Lovelock and Hanno Rund, Tensors, Differential Forms and Variational Principles.
2. Landau L.D. and Lifshitz E.M. Course of Theoretical Physics. Vol. 2. The classical theory of fields.
3. B.A. Dubrovin, S.P. Novikov and A.T. Fomenko. The Modern Geometry. Vol. 1.