Math 488-588

Solitons in Mathematics and Physics

Vladimir Zakharov

Solitons, the nonlinear localized objects, play a very important role in different areas: nonlinear optics, hydrodynamics, plasma theory, superfluidity, and magnetism. Also, they are important for the theory of general relativity – the black holes are solitons. It is remarkable that this broad variety of physical phenomena, from microscopic to astronomic scale, can be described by unified mathematical apparatus that was intensively developed during last four decades. The mathematical theory of solitons, known as the Inverse Scattering method, is closely connected to the spectral theory of differential operators and to the classical theory of integrable Hamiltonian systems.

In this course we will discuss the basic elements of both physics and mathematics of solitons. We will make accent on pure algebraic method for construction of solitonic solutions via local and non-local d-bar problem. Then we will develop the Inverse Scattering method for the Schrodinger and Dirac operators and for their discrete versions. The course will be organized as follow:

1. Basic integrable models in the nonlinear wave dynamics and their interconnection. Lax representations. Gauge equivalence. Simple solitonic solutions of basic equations: KdV, NLSE, N-wave, KP-1, KP-2, sine-Gordon equations. (4 weeks)
2. Method of the local d-bar problem in 1+1 dimensions. Construction of multi-solitonic solutions. Interaction of solitons. Breathers. (3 weeks)
3. Method of the non-local delta-problem. Solitonic solutions depending on functional parameters. Solutions of KP-1 and KP-2 equations. (2 weeks)
4. The method of the Inverse Scattering transform for continuous and discrete Lax operators. Asymptotic behavior of non-solitonic solutions. (3 weeks)
5. Self-similar solutions of integrable systems and Painleve systems. (2 weeks)
6. Solitons in optic fibers (1 week)
7. Solitons in General relativity. Black holes as solitons.

The course will available for undergraduate students with basic knowledge of ODE, linear algebra and some complex analysis. Lecture notes will be posted on internet before each lecture.