Proposal for a Topics Course
Homological Algebra

Romyar Sharifi

This course will provide an introduction to homological algebra: a topic in mathematics of great importance in algebra, geometry, and topology. Homological algebra is an indispensable tool for mathematicians doing research in these areas.

The course will begin with category theory, which serves as a way of comparing structures in objects from different parts of mathematics. It will proceed to homological algebra, which while modeled around the study of chain complexes of modules over rings, works more generally in abelian categories. We will discuss derived functors and their seeming ubiquity in modern mathematics, specific examples of them, and methods of computing them, such as spectral sequences. Towards the end of the course, we will discuss derived categories, which provide a natural setting for comparing cohomology groups of chain complexes.

The only prerequisite for the course is the graduate algebra sequence or its equivalent. The course will use notes prepared by the instructor. The other primary reference is:

C. Weibel, An Introduction to Homological Algebra, Springer.

Topics to be covered, as time permits, include:

- Categories, functors, natural transformations
- Limits, adjoint functors, representability
- Additive and abelian categories
- Chain complexes, homotopy equivalence
- Delta-functors
- Projective and injective objects, derived functors
- Tor/Ext, group cohomology
- Spectral sequences
- Triangulated categories
- Derived categories

Throughout the course, we hope to touch on applications to different areas of mathematics, such as commutative algebra, algebraic geometry, algebraic topology, differential geometry, and algebraic number theory.