Analytical Combinatorics of Random Maps

The goal of this course is to introduce students to an emergent area of research that applies basic methods of analysis, such as complex analysis, asymptotic analysis, probability theory and differential equations, to the study of generating functions associated to combinatorial problems. This is a broad area but I propose to explore it through the lens of a particular class of combinatorial objects known as random maps. The latter are essentially random tesselations of Riemann surfaces. The kinds of questions one first asks about these objects are similar to those one asks about random graphs and, as with that subject, one is led to many intriguing problems of pure and applied mathematical relevance.

The coarse outline I have for the course at this time goes like this:

I. What is analytical combinatorics?
II. A brief introduction to graph theory (both enumerative & random)
III. Basic notions on combinatorial topology
IV. History and tools (combinatorial & analytical) for studying random maps
V. Fundamental results on map enumeration
VI. Selected applications for random map analysis; some possibilities:
    i) Random matrix theory
    ii) Random lattice paths and orthogonal polynomials
    iii) Toeplitz & Hankel operators
    iv) Continuum limits of the above
    v) Discrete geometric analysis
    vi) Statistical mechanics on random lattices
    vii) asymptotic representation theory for finite and compact groups
    viii) infinite dimensional harmonic analysis

The prerequisites for this course are a solid background in the material of the Core courses and an ability/willingness to think about how diverse areas of mathematics are related to one another.