Title: Geometry of Yang-Mills Theory
Instructor: Sergey Cherkis

Yang-Mills theory naturally combines group theory and geometry of manifolds into one subject. It has numerous applications in pure mathematics, such as in the classification of smooth four-manifolds and in knot invariants. In physics it describes all of the fundamental forces of Nature, except gravity.

This course will introduce the basic geometry of Yang-Mills theory. This includes defining and illustrating such concepts as principal bundles, vector bundles and connections on them, gauge invariance, and basic functionals on the space of connections. The main focus of the course will be on solitons in Yang-Mills theory and associated integrable systems. The general theory will be supplemented with numerous concrete examples. The course will be self contained, requiring familiarity with basic analysis and geometry of smooth manifolds. Students will learn modern methods of constructing instantons, monopoles and will be offered open problems that can be addressed by these methods.

Prerequisites: Math 511A, 523A, and 534A.

Texts:

M.F. Atiyah, "Geometry of Yang-Mills Fields," Pisa (1979) http://tinyurl.com/omq943x

N. Hitchin, "Monopoles, Minimal Surfaces, and Algebraic Curves,"
Séminaire de Mathématiques Supérieures, 105. Presses de l'Université de Montréal, Montreal, PQ, 1987. 94 pp. ISBN: 2-7606-0801-8

Syllabus:

- Principal and associated bundles. Connections on them.
- Gauge invariant functionals on the space of connections.
- Instantons, monopoles, vortices as minima of Yang-Mills functional.
- Symplectic formalism and symplectic reduction.
- Moduli spaces and metric on them.
- Construction of instantons by Atiyah, Hitchin, Drinfeld, and Manin.
- Construction of monopoles.
- Monopoles as integrable systems.
- Open problems.