Title: Introduction to Deformation Theory and Applications.

Instructor: Yi Hu

Abstract. This will be an introduction course to deformation theory. We will start with the basic ideas of Kodaira on the deformation of complex manifolds, introducing Kodaira-Spencer maps. We will then focus on the deformation theory of algebraic schemes, introducing Hilbert schemes and Quot schemes of Grothendieck. We will introduce the deformation theory of vector bundles/ coherent sheaves over a fixed scheme. As a result, we hope to furnish students with some good ideas about the local neighborhoods of the moduli spaces of curves/surfaces/varieties and moduli of coherent sheaves over a fixed scheme. Along the way, some basics on the theory of moduli stacks will be explained. But instead of diving into the vast and deep theory of STACKS (see A. J. de Jong's stack project http://stacks.math.columbia.edu/), we will only focus on what are sufficient for the practical moduli problems (namely moduli of schemes and coherent sheaves). Time permits, we will also mention some important applications such as the virtual cycle theory and applications to physicists' mirror symmetry.

Prerequisites: One year course on Algebraic Geometry, e.g., 536A, B or the equivalent. If you only took one semester course on AG (e.g., 536A), please make an appointment and talk to me.

Reference. Fundamental Algebraic Geometry: Grothendieck's FGA Explained Deformation theory of algebraic schemes, Sernesi. Deformation Theory, Hartshorne