Solitons in Mathematics and Physics
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Course Content

Solitons, the nonlinear localized objects, play a very important role in different areas: nonlinear optics, hydrodynamics, plasma theory, superfluidity, and magnetism. Also, they are important for the theory of general relativity – the black holes are solitons. It is remarkable that this broad variety of physical phenomena, from microscopic to astronomic scale, can be described by unified mathematical apparatus that was intensively developed during last four decades. The mathematical theory of solitons, known as the method of Inverse scattering transform, is closely connected to the spectral theory of differential operators and to the classical theory of integrable Hamiltonian systems.

In this course we will discuss the basic elements of both physics and mathematics of solitons. We will make accent on pure elementary algebraic methods for construction of solitonic solutions. Then we will develop the method of Inverse scattering transform for the Schrodinger and Dirac operators. The course will be organized as follow:

1. Basic integrable models in the nonlinear wave dynamics and their interconnection. Lax representations. Gauge equivalence. Simple solitonic solutions of basic equations: KdV, NLSE, N-wave, KP-1, KP-2, sine-Gordon equations (4 weeks)
2. Method of Inverse Scattering Transform for the KdV equation. Riemann-Hilbert problem appears (3 weeks)

3. Method of Inverse Scattering Transform for the Nonlinear Schrodinger equation (2 weeks).

4. Elementary methods for construction of multisolitonic equations (3 weeks).

5. Solitons in optical fibers. Solitons over unstable condensate (2 weeks).

6. Solitons in 2+1 dimensions (2 weeks).

7. Solitons on vortex line and in magnetics (1 week).

The course will available for undergraduate students with basic knowledge of ODE, linear algebra and complex analysis. Prerequisites for the class are MATH 215, 254, 322 or 422. Lecture notes will be posted on internet.