INTRODUCTION TO MICRO-LOCAL ANALYSIS
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Micro-local analysis is a rapidly developing area of mathematics that, back in the 60th and the 70th, revolutionized the theory of linear PDE, the spectral theory of differential operators, provided necessary tools for solving the index problem by Atiyah and Singer, and that has numerous applications to non-linear PDEs, geometry, representation theory, etc.. I am planning the course as an introductory one. I will cover the following topics.

1. Review of the distribution theory. Singularities of distributions. The wave front set of a distribution.
2. Existence of a fundamental solution for a linear PDE with constant coefficients
3. Pseudo-differential operators. Construction of a parametrix for an elliptic linear operator. The index of an elliptic operator.
4. The zeta-function of an elliptic operator and its analytic continuation. The heat trace. Application to the index theory.

Time permitting, I may also cover

5. Fourier integral operators and their application to hyperbolic equations.
6. Propagation of singularities of solutions to a hyperbolic equation.

Prerequisites: Core Math. or Applied Math. courses; some knowledge of PDE, at least at the undergraduate level, will be helpful.