Proposal for special topics course

Hang Xue

Course Title: Topics in Number Theory

Course Description: This course is an introduction to various topics in modern number theory. These topics are usually not covered in a standard algebraic number theory class, but every number theorist knows them. The topics to be covered in this course are the following. These topics are more or less independent.

1. Hasse's local-global principal for quadratic forms.
2. Elliptic curves and Mordell's theorem on the finite generation of the group of rational points, the statement of the BSD conjecture.
3. Rudiments of classical modular forms.

Textbooks: We will use the following textbooks as references. We follow the book by Kato, Kurokawa and Saito for the first two topics and use Serre for the third. We might borrow some material from Silverman or Diamond–Shurman.

1. Kato, Kazuya; Kurokawa, Nobushige; Saito, Takeshi: Number theory. 1. Fermat's dream. Translated from the 1996 Japanese original by Masato Kuwata. Translations of Mathematical Monographs, 186. Iwanami Series in Modern Mathematics. American Mathematical Society, Providence, RI, 2000. xvi+154 pp.
2. Serre, J.-P. A course in arithmetic. Translated from French. Graduate Texts in Mathematics, No. 7. Springer-Verlag, New York-Heidelberg, 1973. viii+115 pp.
3. Diamond, Fred; Shurman, Jerry. A first course in modular forms. Graduate Texts in Mathematics, 228. Springer-Verlag, New York, 2005. xvi+436 pp.
4. Silverman, Joseph H. The arithmetic of elliptic curves. Corrected reprint of the 1986 original. Graduate Texts in Mathematics, 106. Springer-Verlag, New York, 1992. xii+400 pp.

Prerequisites: We do not require much prerequisite for the course. Solid undergraduate background in math should be sufficient. So this class is suitable for *all* graduate students who are interested in number theory. For the most part of the course, familiarity with the notion of groups, rings and fields are assumed. These notions will be used freely throughout

the course. We do not assume Galois Theory. In some rare occasions, e.g. the BSD conjecture and the meromorphic continuation of the Dirichlet L-functions, the notion of analytic functions and meromorphic continuation is required. Some knowledge of point-set topology, e.g. open and closed subsets, convergence, compactness, etc. is not require, but helpful.