Graph Theory

Instructor: David Glickenstein

Overview: Graph theory is a broad area that touches many parts of mathematics and computer science. Euler made one of the earliest uses of graphs to solve the Koenigsberg Bridge Problem, as to whether or not one can tour the city while crossing each bridge once (it turned out you cannot). Sometimes parts of a graph are given attributes, such as resistances on edges or colors on vertices, and these lead to interesting questions that can be succinctly stated. Many of these problems tie to other fields such as:

- Topology: For which surfaces can a graph be embedded without crossing? How is this related to coloring maps of the surface?

- Combinatorics: How can we count various properties of graphs such as the number of spanning trees?

- Optimization: What is the most material that can be transported through a network of pipes (or wires or roads or ...)? How stable is this if some of the pipes fail? How can I efficiently route through the network? Are there types of networks that can be easily traversed and some that cannot?

- Decision theory: How can I rank various types of data given by preferences? Are there stable ways to do this? What if the preferences are not consistent (such as A prefers B, B prefers C, C prefers A)?

Book: The primary text will be *Modern Graph Theory* by B. Bollobás. Supplementary texts may include:

- *Graph Theory* by Bondy and Murty

- *Graph Theory* by Diestel

- The $25,000,000,000 Eigenvector: The Linear Algebra Behind Google by K. Bryan and T. Leise.

- The Small World Phenomenon: An Algorithmic Perspective, by Jon Kleinberg.

- Collective dynamics of 'small-world' networks, by Duncan Watts and Steven Strogatz, published in Nature

- Statistical ranking and combinatorial Hodge theory by Jiang, Lim, Yao and Ye

Description: In this course we will study graph theory from a mathematics perspective, describing some of the more basic definitions and theorems and then look at some more advanced topics. The topics may include:

- Koenigsberg Bridge Problem and Euler characteristic

- Embeddings and genus of a graph, and the Five Color Theorem (and a bit on the Four Color Theorem)

- Hamiltonian cycles and the traveling salesman problem

- The graph Laplacian and the Matrix-Tree Theorem

- Min Flow - Max Cut Theorem

- The PageRank method of ranking

- Stable marriage

- Small world phenomena

- Ramsay theory

- Random graphs

- Combinatorial Hodge theory

Prerequisites: This course requires mathematical maturity at the level of first year graduate courses in the Math or Applied Math core sequence, as the course will be proof based. Strong background in linear algebra is necessary. Some background in topology and differential geometry might be useful, but is not required.