NEW IDEAS IN NONEQUILIBRIUM STATISTICAL MECHANICS, Spring 2017

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Developments that took place in statistical mechanics during the last twenty five years, have transformed the field, leading to new understanding of entropy and its role. It became possible to quantify irreversibility, the most important feature of evolution in statistical mechanics. Even classical thermodynamics acquired a new meaning. We are all familiar with its famous Second Law, which says that entropy of an isolated system does not decrease in time. This statement can now be derived as a consequence of an identity, virtually unknown a quarter of a century ago.

In this course, I want to focus mainly on Langevin equations, which describe an open system—a system interacting with the environment. We are going to study the changes of its energy and entropy. This will lead to *fluctuation relations*; in particular we will derive the famous *Jarzynski equality.* A crucial role in these developments is played by the fact that entropy is a fluctuating quantity, a random variable. The Second Law describes the behavior of its average value. For individual realizations of the system's evolution, entropy does not necessarily increase, but the probability that it does not, decays exponentially with time, at a rate which can be explicitly expressed.

The general theory will be illustrated by examples of physical interest. One of the motivating forces behind the development of the new theory, called *stochastic thermodynamics*, came from applications to small biological systems (bioparticles). In this case entropy fluctuations play an important role and the systems cannot be analyzed using classical thermodynamics.

On the other hand, we will derive classical statements of irreversible thermodynamics, including *Onsager reciprocity relations* and the *principle of minimum entropy production* from the stochastic dynamics of Langevin equation, showing that the new theory contains the old one.

Mathematical language of nonequilibrium statistical mechanics are stochastic differential equations (SDE) I will develop SDE theory from scratch; rather than presenting it as an abstract field, the course will show it in action, applied to describe evolution of open systems.

A rough plan is as follows:

1. Introduction to nonequilibrium statistical mechanics. Langevin equations.

2. Basic Itô stochastic calculus.

3. Linear equations with dissipation. Stationary states.

4. Equilibrium states and reversibility.

5. Hamiltonian equations with damping and noise.

6. Hatano-Sasa and Jarzynski fluctuation relations.

7. Fundamental expression for entropy production.

8. Irreversibility, entropy production and Onsager's reciprocity relations.

9. Applications, including Landauer inequality.

10. Overdamped systems and entropy production.

I am not assuming any prior knowledge of stochastic processes or statistical mechanics. Elementary probability and differential equations will be helpful. While the course material can be presented as rigorous mathematical physics, I will not strive for completeness of all proofs, focusing on ideas behind them and on physical content of the theory. It is a beautiful, exciting field and I want to share it and study it together with graduate students. The main sources for the course material are the articles:

1. R. Chetrite, K. Gawędzki: Fluctuation relations for diffusion processes. Commun. Math. Phys. 282 (2), 469-518.

2. K. Gawędzki: Fluctuation relations in stochastic thermodynamics. http://arxiv.org/pdf/1308.1518.pdf

3. R. E. Spinney, I. J. Ford: Fluctuation relations: a pedagogical overview, arXiv:1201.6381

4. U. Seifert: Stochastic thermodynamics, fluctuation theorems, and molecular machines, Rep. Prog. Phys. 75 (2012)

Nonequilibrium statistical mechanics is one of my research areas and open problems, including dissertation projects, will be discussed towards the end of the semester.

See you in January!