Special topics course:

Mathematical foundations of data assimilation and inverse problems

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Many problems in science and engineering require that one merges mathematical/numerical models with data: numerical weather models are re-calibrated to weather data (temperature, winds, humidity) every six hours to make reliable forecasts for the next six hours; reservoir models are tuned to flow rate measurements to yield accurate representations of subsurface flows; surface velocity data of the polar ice caps are used to estimate friction coefficients at the interface of ice, water and rock. The goal of this class is to explain how algorithms that perform the task of merging models and data function.

The class will start with a brief (1–2 weeks) review of random variables, conditional probability, and Monte Carlo sampling. We will then discuss the Bayesian approach to merging models with data, called "data assimilation" (DA). The Kalman filter will be presented as a classic DA tool, and to motivate the ensemble Kalman filter (EnKF) as a Monte Carlo approximation of the Kalman filter. We discuss EnKF's limitations and move on to nonlinear/non-Gaussian methods (particle filters) and their limitations. I will then introduce optimization techniques for solving data assimilation problems and make connections between the variational and Bayesian approaches to data assimilation. In the last few weeks I will introduce Markov Chain Monte Carlo (MCMC) and its application to Bayesian inverse problems. Throughout the semester we will code up most of the DA methods we study and we will use the Lorenz'96 model as our prototypical numerical model.

Prerequisites: Since part of the class and homework assignments will involve coding, basic programming experience is necessary (any language, Matlab, C, python etc. is fine). Basic knowledge of (numerical) ODE and optimization will be assumed.

Books, notes and references:

1. A.J. Chorin and O. Hald, *Stochastic Tools in Mathematics and Science*, 3rd Edition, Springer, 2013.

2. S. Reich and C. Cotter, *Probabilistic Forecasting and Bayesian Data Assimilation*, Cambridge Texts in Applied Mathematics, 2015.

3. M.S. Arulampalam, S. Maskell, N. Gordon, T. Clapp, *A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking*, IEEE Transactions on Signal Processing **50**(2), 2002.

4. Art B. Owen, *Monte Carlo theory, methods and examples*, 2013

5. D.J.C. Mackay, *Introduction to Monte Carlo Methods*, Learning in Graphical Models, Vol. 89 of the series NATO ASI Series, pp. 175–204.

6. A. Tarantola, *Inverse Problem Theory and Methods for Parameter Estimation*, SIAM, 2005.