Special topics course:

Mathematical foundations of data assimilation and inverse problems- Part II

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Many problems in science and engineering require that one merges mathematical/numerical models with data: numerical weather models are re-calibrated to weather data (temperature, winds, humidity) every six hours to make reliable forecasts for the next six hours; reservoir models are tuned to flow rate measurements to yield accurate representations of subsurface flows; surface velocity data of the polar ice caps are used to estimate friction coefficients at the interface of ice, water and rock. The goal of this class is to explain how algorithms that perform the task of merging models and data function.

Part I of this class will included a brief (2 weeks) review of random variables, conditional probability, and Monte Carlo sampling. We then discussed the Bayesian approach to merging models with data, called "data assimilation" (DA), and we covered several numerical methods for data assimilation. This includes the Kalman filter, the ensemble Kalman filter (EnKF), and variational methods. We started to discuss importance sampling as a method for drawing samples from arbitrary probability densities.

In Part II of this class we continue to study importance sampling and their application in data assimilation (particle filters). We will discuss the limitations of this approach and how these limitations can be overcome for a certain problem class by exploiting sparsity of covariance matrices (localization). We will then move on to "inverse problems", in which parameter and state estimates are not updated as frequently. As in Part I of this class, an important aspect of the class is to code up the various numerical methods, using the Lorenz'96 model as our prototypical numerical model. We will also explore other model (predator-prey and some simple PDE) in parameter estimation problems and we will also assimilate "real" data.

Prerequisites: Since part of the class and homework assignments will involve coding, basic programming experience is necessary (any language, Matlab, C, python etc. is fine). Basic knowledge of (numerical) ODE and optimization will be assumed. For Part II of this class, Part I is a requirement.

Books, notes and references:

1. A.J. Chorin and O. Hald, *Stochastic Tools in Mathematics and Science*, 3rd Edition, Springer, 2013.

2. S. Reich and C. Cotter, *Probabilistic Forecasting and Bayesian Data Assimilation*, Cambridge Texts in Applied Mathematics, 2015.

3. M.S. Arulampalam, S. Maskell, N. Gordon, T. Clapp, *A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking*, IEEE Transactions on Signal Processing **50**(2), 2002.

4. Art B. Owen, *Monte Carlo theory, methods and examples*, 2013

5. D.J.C. Mackay, *Introduction to Monte Carlo Methods*, Learning in Graphical Models, Vol. 89 of the series NATO ASI Series, pp. 175–204.

6. A. Tarantola, *Inverse Problem Theory and Methods for Parameter Estimation*, SIAM, 2005.