Math 488-588

Solitons in Mathematics and Physics

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Course Content

Theory of solitons is a relatively new and fast growing branch of mathematical physics. Its development leads to progress in such areas of pure mathematics as spectral theory of differential operators, complex algebraic geometry, and classical theory of integrable systems. Solitons, the nonlinear localized objects, play very important role in different areas of physics: nonlinear optics, hydrodynamics, plasma theory, superfluidity, and magnetism. Also, they are important for the theory of general relativity: the black holes are solitons. It is remarkable that this broad variety of physical phenomena, from microscopic to astronomic scale, can be described by unified mathematical apparatus. The mathematical theory of solitons employs a combination of several powerful methods: the method of inverse scattering transform, the dressing method, the direct Hirota method, the method of N-gap integration.

In this course we will discuss the basic elements of both mathematics and physics of solitons, will make an accent on pure elementary algebraic methods for construction of solitonic solutions, and will develop the method of inverse scattering transform for the Schrodinger and Dirac operators. The course will be organized as follows:

1. Basic integrable models in the nonlinear wave dynamics and their interconnection. Lax representations. Gauge equivalence. Simple solitonic solutions of basic equations: KdV, NLSE, N-wave, KP-1, KP-2, sine-Gordon equations (4 weeks)
2. Elementary methods for construction of multisolitonic equations (3 weeks)
3. Method of inverse scattering transform for the KdV equation. Riemann-Hilbert problem appears (3 weeks)
4. Method of inverse scattering transform for the Nonlinear Schrodinger equation (2 weeks)
5. Solitons in optical fibers. Solitons over unstable condensate (2 weeks)
6. Solitons in 2+1 dimensions (2 weeks)
7. Solitons on vortex line and in magnetics (1 week)

Lecture notes will be posted online before each lecture.

The course is intended for the graduate students but will be available for the determined undergraduates with basic knowledge of ODE, linear algebra and complex analysis. During the second part of the course the students will be offered individual scientific projects, which potentially can become a foundation for scientific publication.
Prerequisites for the class are MATH 215, 254, 322 or 422.