Riemann-Hilbert Problems and Their Applications

Instructor: Nick Ercolani

Overview: Riemann-Hilbert Problems arose as a synthesis of classical methods in complex function theory (in particular the analysis of contour integrals) and Fourier analysis. Their principal classical applications were to asymptotic questions related to linear systems of differential equations with singularities and the theory of special functions. A recent resurgence of interests in these problems has stemmed from a confluence of developments in nonlinear PDEs (partial differential equations), lattice spin systems, integrable systems theory and their asymptotic analysis. This has brought with it a whole new range of applications to other, often new, fields: random matrix theory, nonlinear special functions, quantum gravity, non-classical approximation theory and spectral theory of Schrodinger operators, to name a few. The purpose of this course will be to develop the basics of Riemann-Hilbert analysis from a modern perspective and explore a few of the above-mentioned motivations and applications.

Description: I envision the course proceeding in four stages:

I. Some necessary background in Complex Analysis.
II. Classical examples of Riemann-Hilbert Problems (RHPs)
III. Fundamental Techniques for solving a general class of RHPs (with examples)
IV. Modern Applications (I have in mind here applications to Random Matrix Theory & Inverse Scattering for Dirac/Schrodinger Spectral Problems; however, topics may vary depending on student interest.)

Texts: The material for parts I – III of the course will draw on a number of sources including

- *Orthogonal Polynomials and Random Matrices: A Riemann-Hilbert Approach* by Percy Deift
- *Riemann–Hilbert Problems, Their Numerical Solution, and the Computation of Nonlinear Special Functions* by Thomas Trogdon and Sheehan Olver

Part IV will largely involve the Instructor's notes.

Prerequisites: A solid background in the material of the core analysis courses (Math 523 or Math 527).

Expected Learning Outcomes:

- A deeper appreciation of the connections between real and complex analysis and facility in passing between the two subjects in solving problems.
- An introduction to a range of current active research areas in Nonlinear Analysis, Mathematical Physics and Combinatorics for which Riemann-Hilbert problems are an essential tool.