Discrete Geometry and Geometric Graph Theory

Instructor: David Glickenstein

Overview: Discrete geometry has become an important are of mathematics with applications to computer graphics, modelling, brain imaging, and many other fields. Given a surface in three dimensions, finding a canonical parametrization that can be used to parametrize the surface is a fundamental problem, with applications to being able to compare different surfaces for a variety of reasons such as statistical analysis or texture mapping. This course will focus on methods of discrete conformal geometry, additionally making connections to graph theory and Riemannian geometry.

Book: The course will be taught out of notes that the instructor is developing. Secondary sources and readings will be drawn from:

- Circle Packing by Ken Stephenson
- Combinatorial Ricci flows by Chow and Luo
- Notes on discrete conformal mapping by F. Luo.
- Statistical ranking and combinatorial Hodge theory by Jiang, Lim, Yao and Ye

Description: In this course we will study graph theory from a mathematics perspective, describing some of the more basic definitions and theorems and then look at some more advanced topics. The topics may include:

- Euclidean, hyperbolic, spherical, and Riemannian geometry (2 weeks)
- Introduction to polyhedral and triangulated surfaces, including theory of Delaunay triangulations (2 weeks)
- Uniformization in Riemannian geometry (1 week)
- Uniformization in discrete geometries (3 weeks)
- Circle packing and Thurston's formulation of conformal mapping (2 weeks)
- Discrete Laplacians and finite element and finite volumes methods (2 weeks)
- The graph Laplacian, Matrix-Tree Theorem, and Tutte embedding of a graph (1-2 weeks)
- Combinatorial Hodge theory (1-2 weeks)

Prerequisites: This course requires mathematical maturity at the level of first year graduate courses in the Math or Applied Math core sequence, as the course will be proof based. Strong background in linear algebra is necessary. Some background in topology, differential geometry, and PDE would be useful, but is not required.

Expected learning outcomes:

- Be able to describe several theorems about the structure of smooth and polyhedral surfaces through conformal deformation.
- Be able to relate certain concepts in graph theory to those in differential geometry and/or complex analysis in a precise way.
- Be able to present these ideas in a clear way with careful definitions and some proofs.