COMPUTABILITY AND LOGIC—A TOPICS COURSE PROPOSAL

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The course will discuss mathematical logic from the point of view of computation theory: algorithms, Turing machines and complexity. Its core will be a detailed treatment of fundamental classical theorems, culminating in the celebrated Gödel Incompleteness Theorem. Time permitting, I will also outline Chaitin's complexity approach to provability and incompleteness and discuss the P and NP classes. I am not assuming any prior knowledge of the subject, just basic mathematical culture at the level of Mathematics 323—and a keen interest in the subject. Advanced undergraduate students are welcome, as are students from other departments.

Sources

The main text will be the book:

G. Boolos, J. Burgess, R. Jeffrey: Computability and logic. Cambridge University Press (2012).

We will cover the first 18 chapters. Discussion of complexity will use several texts by G. Chaitin and that of the P-NP problem will be based on articles by A. Wigderson.

Learning outcomes

After taking the course, the students will be familiar with the fundamental concepts of mathematical logic, starting from elementary, but leading to advanced, including Gödel Incompleteness Theorem, the most important result in the field. They will also be exposed to basic notions of computability theory: Turing machines, recursive functions, algorithmic complexity. A discussion of the P-NP conjecture at the end of the course will introduce one of the currently central topics in the field.

Schedule

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1. Turing machines and Turing computability.
2. Recursive functions and Church's thesis.
3. First-order logic
4. Model theory
5. Proof theory
6. Arithmetization of logic
7. Incompleteness and undecidability theorems
8. Introduction to algorithmic complexity
9. Introduction to the P-NP conjecture