The following problems cover the skills that are necessary to be successful on Test A.

1. Simplify: \( \sqrt{\frac{-16x^3}{2y^6}} \).

\[
\begin{align*}
\sqrt{\frac{-16x^3}{2y^6}} &= \left( \frac{-16x^3}{2y^6} \right)^{\frac{1}{2}} \\
&= \left( \frac{-16}{2} \right)^{\frac{1}{2}} \cdot (x)^{\frac{3}{2}} \cdot \left( \frac{1}{y^3} \right)^{\frac{1}{2}} \\
&= (-8)^{\frac{1}{2}} \cdot x^{\frac{3}{2}} \cdot \left( \frac{1}{y^3} \right)^{\frac{1}{2}} \\
&= -2 \cdot x \cdot \frac{1}{y^2} \\
&= -\frac{2x}{y^2}
\end{align*}
\]

2. Perform the indicated operations and simplify: \((m^{n+1}r^n)(3m^nr^{2n})^{-1}\).

\[
\begin{align*}
(m^{n+1}r^n)(3m^nr^{2n})^{-1} &= \frac{m^{n+1}r^n}{3m^nr^{2n}} \\
&= \frac{m^{n+1}}{3m^n} \cdot \frac{r^n}{r^{2n}} \\
&= \frac{m}{3} \cdot \frac{r^n}{r^n} \\
&= \frac{m}{3}
\end{align*}
\]

3. Perform the indicated operations and simplify: \( \frac{ab}{\frac{1}{a} + \frac{1}{b}} \).

\[
\begin{align*}
\frac{ab}{\frac{1}{a} + \frac{1}{b}} &= \frac{ab}{\frac{b}{ab} + \frac{a}{ab}} \\
&= \frac{ab}{\frac{a+b}{ab}} \\
&= \frac{a^2b^2}{a+b} \\
&= \frac{a^2b^2}{b+a}
\end{align*}
\]

4. Rationalize the denominator: \( \frac{2}{\sqrt{2} + b} \).

\[
\frac{2}{\sqrt{2} + b} \cdot \frac{\sqrt{2} - b}{\sqrt{2} - b} = \frac{2(\sqrt{2} - b)}{2 - b^2}
\]

5. Evaluate \((5x+1)^{\frac{3}{4}} - (7-x)^0\) for \(x = 3\).

\[
\begin{align*}
(5x+1)^{\frac{3}{4}} - (7-x)^0 &= (5(3)+1)^{\frac{3}{4}} - (7-(3))^0 \\
&= (15+1)^{\frac{3}{4}} - (4)^0 \\
&= (16)^{\frac{3}{4}} - 1 \\
&= 8 - 1 = 7
\end{align*}
\]
6. Evaluate \((-2b^2)^{-1}\) when \(b = -2\).

\[-(2b^1)^{-1} = -(2(-2)^3)^{-1} = -(2(4))^{-1} = -\frac{1}{8}\]

7. Simplify completely:

\[2\sqrt{50} - 7\sqrt{18 + \sqrt{8}}\]

\[= 2\sqrt{25 \cdot 2} - 7\sqrt{9 \cdot 2 + \sqrt{4 \cdot 2}}\]

\[= 2 \cdot 5\sqrt{2} - 7 \cdot 3\sqrt{2} + 2\sqrt{2}\]

\[= 10\sqrt{2} - 21\sqrt{2} + 2\sqrt{2}\]

\[= -9\sqrt{2}\]

8. Simplify completely:

\[2u(3u^2 - 1) - (-8u^3 - 14u + 6)\]

\[= 2u(3u^2) - 2u(1) + 8u^3 + 14u - 6\]

\[= 6u^3 - 2u + 8u^3 + 14u - 6\]

\[= 14u^3 + 12u - 6\]

9. Simplify completely:

\[4(2x+1)^2 + 3(2x+1) + 1\]

\[= 4(2x+1)(2x+1) + 3(2x+1) + 1\]

\[= 4(4x^2 + 2x + 2x + 1) + 6x + 3 + 1\]

\[= 4(4x^2 + 4x + 1) + 6x + 4\]

\[= 16x^2 + 8x + 4 + 6x + 4\]

\[= 16x^2 + 14x + 8\]

10. Factor completely:

\[32x^4y - 162y\]

\[= 2y(16x^4 - 81)\]

\[= 2y(4x^2 + 9)(4x^2 - 9)\]

\[= 2y(4x^2 + 9)(2x + 3)(2x - 3)\]

11. Perform the indicated operation and simplify completely:

\[\frac{z^2 + z - 12}{2z^2 + 6z} \cdot \frac{z^2 + 3z}{6z + 24}\]

\[= \frac{(z - 3)(z + 4)}{2z(z + 3)} \cdot \frac{z(z + 3)}{6(z + 4)}\]

\[= \frac{z - 3}{2z} \cdot \frac{z}{6} = \frac{z - 3}{12z}\]
12. Perform the indicated operation and simplify: \( \frac{3c}{c-2} + \frac{c+1}{2-c} \).

\[
\frac{3c}{c-2} + \frac{c+1}{2-c} = \frac{3c}{c-2} - \frac{c+1}{(c-2)} = \frac{3c - (c+1)}{(c-2)} = \frac{2c - 1}{c-2}.
\]

13. Solve for \( z \):

\[
7z - (4z - 9) = 24 + 5(z - 1)
\]

\[
= 7z - 4z + 9 = 24 + 5z - 5
\]

\[
= 3z = 19 + 5z
\]

\[
= -10 = 2z
\]

\[
-5 = z
\]

14. Solve for \( x \):

\[
\frac{a}{3} + 5x = b\left(\frac{x}{3} + 2\right)
\]

\[
3 \cdot \left(\frac{a}{3} + 5x = \frac{bx}{3} + 2b\right) \cdot 3
\]

\[
= a + 15x = bx + 6b
\]

\[
= 15x - bx = 6b - a
\]

\[
= x(15-b) = 6b - a
\]

\[
x = \frac{6b - a}{15-b}
\]

15. Solve for \( t \):

\[
2t^2 + 4t = 9t + 18.
\]

\[
2t^2 + 4t - 9t - 18 = 0
\]

\[
= 2t^2 - 5t - 18 = 0
\]

\[
= (2t-9)(t+2) = 0
\]

\[
t = \frac{9}{2}, -2
\]

16. Solve for \( s \):

\[
-2s^2 - 4s + 2s^3 = 0
\]

\[
= 2s(-s^2 - 2 + s^2) = 0
\]

\[
= 2s(s^2 - s - 2) = 0
\]

\[
= 2s(s-2)(s+1) = 0
\]

\[
s = 0, s = 2, s = -1
\]

\[
S = 0, 2, -1
\]
17. Solve for \( p \):
\[
\frac{4}{p} - \frac{2}{p+1} = 3.
\]

\[
p \left(\frac{4}{p} - \frac{2}{p+1}\right) = 3(p)
\]
\[
= 4 - \frac{2p}{p+1} = 3p(p+1)
\]
\[
= 4(p+1) - 2p = 3p(p+1)
\]
\[
= 4p + 4 - 2p = 3p^2 + 3p
\]
\[
= 2p + 4 = 3p^2 + 3p
\]
\[
3p^2 + p - 4 = 0
\]
\[
(3p + 4)(p - 1) = 0
\]
\[
p = -\frac{4}{3}, 1
\]

18. To get a B in a course a student must have an average of at least 80% on five tests that are worth 100 points each. On the first four tests a student scores 92%, 83%, 61%, and 71%. Determine the lowest score the student can receive on the fifth test to assure a grade of B for the course.

\[
80 = \frac{0.92 + 0.83 + 0.61 + 0.71 + x}{5}
\]
\[
4.00 = 3.07 + x
\]
\[
0.93 = x
\]

The student must get at least a 93% on the fifth test to receive a B in the class.

19. The area of a rectangle is 84 square feet and the length is 6 feet longer than the width. If \( w \) represents the width, write an equation that could be used to find the dimensions of the rectangle.

\[
A = l \cdot w = 84 \text{ ft}^2
\]
\[
l = w + 6
\]
\[
84 = (w + 6) \cdot w
\]
\[
w(w + 6) = 84
\]

20. A furniture store drops the price of a table 37 percent to a sale price of $364.77. What is the original price?

Let \( P \) represent the original price.

\[
P - 0.37P = 364.77
\]
\[
P(1-0.37) = 364.77
\]
\[
P(0.63) = 364.77
\]
\[
0.63 \cdot 0.63 \Rightarrow P = 579
\]

21. Solve for \( t \):
\[
(t+2)^2 = 8
\]
\[
\sqrt{(t+2)^2} = \sqrt{8}
\]
\[
t+2 = \pm\sqrt{8}
\]
\[
t = -2 \pm 2\sqrt{2}
\]
\[
t = -2 \pm 2\sqrt{2}
\]

22. Solve for $z$: $z^2 - 4z + 6 = 0$.

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 1 \quad b = -4 \quad c = 6$$

$$z = \frac{4 \pm \sqrt{16 - 24}}{2} = \frac{4 \pm \sqrt{-8}}{2} = \frac{4 \pm 2i\sqrt{2}}{2} = 2 \pm i\sqrt{2} \rightarrow z = 2 \pm i\sqrt{2}$$

23. Perform the indicated operation and simplify: $\sqrt{-2} \cdot \sqrt{-24}$.

$$\sqrt{-2} \cdot \sqrt{-24} = \sqrt{-(2)(24)} = \sqrt{-48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$$

24. Solve for $r$: $5 - 3r \leq 8$.

$$5 - 3r \leq 8$$

$$-3r \leq 3$$

$$r \geq -1$$

25. Solve for $x$: $|2x + 1| \geq 7$.

$$|2x + 1| \geq 7$$

$$-(2x + 1) \geq 7 \quad \text{or} \quad (2x + 1) \geq 7$$

$$-2x - 1 \geq 7$$

$$2x \geq -8$$

$$x \leq -4$$

$$2x \geq 6$$

$$x \geq 3$$

$$(-\infty, -4] \quad \text{or} \quad [3, \infty)$$

26. Find the domain of $y = \sqrt{4 - 5x}$.

$$0 \leq \sqrt{4 - 5x}$$

$$0 \leq 4 - 5x$$

$$5x \leq 4$$

$$x \leq \frac{4}{5} \quad \rightarrow \quad [-\infty, \frac{4}{5}]$$

27. Find the $x$-intercepts of $y = -2x^2 - 13x + 6$.

$$y - 2x^2 - 13x + 6 = 0$$

$$2x^2 + 13x + 6 = 0$$

$$(2x + 1)(x + 6) = 0$$

$$x = -\frac{1}{2} \quad x = -6$$

$$(-\frac{1}{2}, 0), (-6, 0)$$
28. Find the equation of the graph at the right:

\[ y = mx + b \]

\( m \) is slope
\( b \) is \( y \)-intercept

We can see that it intersects the \( y \)-axis at \( y = 2 \) so \( b = 2 \).

\[ m = \frac{\text{rise}}{\text{run}} = \frac{4}{4} \quad \text{and} \quad \frac{y}{-4} = -1 \]

Because of the vertex on the \( y \)-axis, we can see that we are taking the absolute value of \( x \), so the equation is: \( y = |x| + 2 \)

29. Find the distance between \((6,3)\) and \((-2,4)\).

\[ \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(2-6)^2 + (4-3)^2} = \sqrt{16+1} = \sqrt{17} \]

30. Find the midpoint of the line segment joining \((6,9)\) and \((-3,1)\).

\[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{6-3}{2}, \frac{9+1}{2} \right) = \left( \frac{3}{2}, \frac{10}{2} \right) = \left( \frac{3}{2}, 5 \right) \]

31. Find the slope and \( y \)-intercept of the line \( 5x + 4y = 8 \).

\[ 5x + 4y = 8 \quad \rightarrow \quad 4y = -5x + 8 \quad \rightarrow \quad y = -\frac{5}{4}x + 2 \]

\( y = mx + b \) where \( m \) is the slope and \( b \) is the \( y \)-intercept, so the slope is \( m = -\frac{5}{4} \) and the \( y \)-intercept is \( b = 2 \).

32. Find the equation of the line perpendicular to \( 3y + 2x - 3 = 0 \) passing through \((4,-1)\).

\( 3y + 2x - 3 = 0 \)

\[ 3y = -2x + 3 \]

\[ y = -\frac{2}{3}x + 1 \]

Perpendicular line slope = \( \frac{3}{2} \)

\[ y - (-1) = \frac{3}{2}(x - 4) \]

\[ y = \frac{3}{2}x - 6 - 1 \quad \rightarrow \quad y = \frac{3}{2}x - 7 \]

\[ 2y = 3x - 14 \]

\[ 2y - 3x + 14 = 0 \]
33. Find \( f(-4) \) if \( f(x) = \frac{2x^2 - 11}{3x} \)

\[
 f(x) = \frac{2x^2 - 11}{3x} \quad \Rightarrow \quad f(-4) = \frac{2(-4)^2 - 11}{3(-4)} = \frac{2(16) - 11}{-12} = \frac{32 - 11}{-12} = -\frac{21}{12} = -\frac{7}{4}
\]

34. Find \( f(b+2) \) if \( f(x) = 5 - 3(x+1) \).

\[
 f(x) = 5 - 3(x+1) \\
f(b+2) = 5 - 3((b+2)+1) \\
f(b+2) = 5 - 3(b+3) \\
f(b+2) = 5 - 3b - 9 \\
\]

\[
f(b+2) = -3b - 4
\]

35. Find the domain and the range of the function graphed at the right:

The domain is the inputs, or \( x \)-values, and the range is the outputs, or \( y \)-values.

**domain:** \([-3, 4]\]  
**range:** \([-2, 2]\]  

36. If \((5,6)\) is a point on the graph of \( y = g(x) \), find a point on the inverse graph, \( g^{-1}(x) \)

\[
y = g(x) \quad \Rightarrow \quad 6 = g(5) \quad \Rightarrow \quad 5 = g^{-1}(6)
\]

\[
(6,5)
\]

37. If \( h(t) = \frac{t}{t+1} \), find the value of \( t \) so that \( h(t) = 3 \).

\[
h(t) = \frac{t}{t+1} \quad \Rightarrow \quad h(t) = 3 \quad \Rightarrow \quad 3 = \frac{t}{t+1}
\]

\[
(t+1) 3 = \frac{t}{t+1} (t+1)
\]

\[
3(t+1) = t
\]

\[
3t + 3 = t
\]

\[
3 = -2t
\]

\[
-\frac{3}{2} = t
\]
38. If the graph of \( y = f(x) \) is at the right, sketch the graph of \( y = |f(x)| \).

When we take the absolute value of a negative number, it becomes positive, so wherever the y-value is negative on the original graph, it is positive on the new graph.

39. Rewrite \( 10^b = a \) in logarithmic form.

\[
\begin{align*}
10^b &= a \\
\log_{10}(10^b) &= \log_{10}(a) \\
b &= \log_{10} a \\
\log a &= b
\end{align*}
\]

40. Rewrite as a single logarithm: \( \frac{1}{2} \log x + 4 \log y - 2 \log z \).

\[
\begin{align*}
\frac{1}{2} \log x + 4 \log y - 2 \log z &= \log \left( x^{\frac{1}{2}} \right) + \log \left( y^4 \right) - \log \left( z^2 \right) \\
&= \log \left( \frac{x^{\frac{1}{2}} y^4}{z^2} \right) \\
&= \log \left( \sqrt{x} \cdot y^4 \cdot \frac{1}{z^2} \right)
\end{align*}
\]

41. Solve for \( t \): \( 3^{2t} = 27^{2t-1} \).

\[
\begin{align*}
3^{2t} &= 3^{3(2t-1)} \\
3^{2t} &= 3^{3t(2t-1)} \\
2t &= 3(2t-1) \\
2t &= 6t - 3 \\
+3 &+3 \\
2t+3 &= 6t \\
-2t &= -2t \\
3 &= 4t
\end{align*}
\]
42. Solve the system of equations:
\[
\begin{align*}
4x + 3y &= 0 \\
8x &= 9y + 2
\end{align*}
\]
\[
\begin{align*}
4x + 3y &= 0 \\
8x - 9y &= 2
\end{align*}
\]
\[
\begin{align*}
(4x + 3y = 0) \cdot 3 &\rightarrow 12x + 9y = 0 \\
(8x - 9y = 2) &\rightarrow (8x - 9y = 2)
\end{align*}
\]
\[
\begin{align*}
6x &= -2 \\
8x - 9y &= 2
\end{align*}
\]

\[
\begin{align*}
-15y &= 2 \\
-15 &\rightarrow y = \frac{2}{15}
\end{align*}
\]

43. Express the length of side \(a\) in terms of \(m\):
\[
\begin{align*}
a^2 + m^2 &= 8^2 \\
a^2 + m^2 &= l^2 + 4 \\
a^2 &= l^2 - m^2
\end{align*}
\]
\[
a = \sqrt{l^2 - m^2}
\]