Enhancing the Mathematical Sciences Workforce

Department of Mathematics and Program in Applied Mathematics

University of Arizona

1 Introduction

The Department of Mathematics and the Program in Applied Mathematics at the University of Arizona propose a far-reaching workforce enhancement program that will prepare young American men and women for careers as professional mathematicians. The mathematical sciences workforce of the 21st century must be diverse not only with respect to gender and ethnicity but also with respect to training to ensure that it can serve the full spectrum of societal needs in mathematical research and education. Furthermore, in order to be truly effective at the national level such a training program must reach far beyond local institutional boundaries—both geographically and culturally—in order to maximize its impact on the growth and development of the emergent professional community. *We will achieve this by integrating the University of Arizona's unique intellectual environment in the mathematical sciences with an innovative summer program which will disseminate our professional culture over a broad geographical region and, at the same time, enhance the flow of talented students from diverse backgrounds to the University of Arizona and other Research 1 institutions.*

Our institution offers:

- A unique and collegial institutional structure of a disciplinary Department of Mathematics and an independent interdisciplinary Program in Applied Mathematics that has consistently generated research and education of the highest quality in the mathematical sciences.
- World-class research programs in many areas of mathematics and its applications ranging from mathematical biology to nonlinear optics, and number theory to partial differential equations.
- A proven track record in innovative mathematics education and outreach activities, including award-winning undergraduate courses, unique laboratory training at the graduate and advanced undergraduate levels, and nationally prominent research-based training at the graduate and post-doctoral levels.
- A highly ranked Ph.D.-granting Mathematics Education program that is a leader in research on K-12 teaching and learning with an emphasis on socio-cultural perspectives, equity, and the use of student thinking in teacher education.

The University of Arizona's location in the Southwestern United States puts it at the heart of many Hispanic and Native American communities and the educational institutions that serve them. Our goal is to create a hub of workforce enhancement activities in the Southwest of a bi-directional nature; namely a structure that brings students *and faculty* from these institutions to Arizona to inform us about educating and mentoring students from diverse backgrounds, and exports our approaches to mathematics education and research back to those institutions in ways that will benefit them and enhance the pipeline of talented students to our own and other schools.

The combined intellectual and institutional resources of the Mathematics Department and the Program in Applied Mathematics make it possible for us to offer an unusually broad spectrum of intellectual and professional development opportunities including: a wide range of high-quality courses for both undergraduates and graduate students; two unique experimental laboratories used for training at the interface of

mathematics and experimentation; a host of research seminars and colloquia; disciplinary and interdisciplinary research opportunities for undergraduates, graduate students, and post-docs; national laboratory and industrial internships; and cross-disciplinary collaborations with faculty in other departments and the Mathematics Education program. Complementing these research and educational opportunities is a wide range of instructional training activities, including carefully supervised classroom teaching, individual and group tutoring, and many mentoring and outreach activities.

During our first VIGRE grant, we developed a variety of individual and team-based mentoring mechanisms to help students make the many transitions necessary for their personal and intellectual development into self-motivated and creative researchers, teachers, communicators, and team players. At the undergraduate level these transitions are effected by a combination of good advising structures and a variety of capstone experiences for seniors, often in a vertically integrated setting. To facilitate the transition from undergraduate life to the rigors of graduate school, the Department of Mathematics and the Program in Applied Mathematics have developed a variety of support and assessment mechanisms to integrate their new graduate students into the intellectual and cultural life of their respective degree programs, stimulate their appetite for enquiry-based learning, and to reduce attrition. Their professional and intellectual development at the intermediate and advanced graduate levels is further developed by the very successful system (developed under our first VIGRE grant) of competitively awarded fellowship support in which students are required to write and submit proposals mapping out the intended use of the funds to fulfill their research and educational objectives. At the post-doctoral level we will continue to provide careful faculty mentoring and oversight to further enhance their professional training for careers in the mathematical sciences. Overall, our teaching and training paradigm is one in which vertical integration between different academic levels and horizontal integration across different fields of mathematics is well mentored, collegial, natural, and flexible—allowing for interactions between groups and individuals at those transition points in their careers when such activities will be most beneficial to them. All our current and proposed activities are designed to be authentic experiences in research, education, and communication that integrate naturally with the participants' scholarly development—thereby creating organizational and motivational structures that will be highly sustainable beyond the lifetime of the grant.

We believe that this model and culture of integrative and interdisciplinary research and education in the mathematical sciences is one that could be profitably spread beyond our institutional boundaries in ways that will make an impact, far greater than the dollar value of grant, on the development of the professional workforce. This will be achieved through an innovative summer program that will not only provide participating students with stimulating research experiences that reflect our particular culture, but also support the development of faculty and programs at a network of regional schools, many of which serve large minority communities and which may lack the institutional infrastructure necessary to compete for, and support, VIGRE-like programs. While most undergraduate summer programs usually operate in an institutionally localized sense—in that the program is run at the host institution by local faculty—our model will involve faculty from schools in our regional network with the goal of not only informing one another of the academic requirements and styles of our respective programs, but also of assisting our partners develop complementary programs at their own institutions. At the same time we look forward to working with, and learning from, some of the established summer programs around the country to ensure the integration of established best practices into our efforts.

Our post-VIGRE plan, which involves funds from private donors and a very substantial financial commitment from the University of Arizona in the form of ten, post-grant, fellowships insures that the needed resources and infrastructure will be in place to continue these activities beyond the period of VIGRE funding. These post-VIGRE sources have been carefully planned to phase in over the term of this VIGRE grant, leading to a smooth transition from the NSF-funded project to a permanent and effective training program that will produce a broadly trained workforce of mathematical scientists who are well-prepared for the challenges of the 21st century.

2 Results from Prior Support

2.1 Graduate Programs

Our previous VIGRE grant supported graduate students via fellowships and travel funds. We begin our discussion of results by explaining the uses to which fellowship and travel support were put. Then we turn to a more global discussion of the impact of VIGRE on our program as a whole. Finally, we round out the discussion with a series of vignettes which illustrate the types of students we produce and how VIGRE support affected their trajectories.

2.1.1 Fellowship and travel support

As we described in the main project description, students obtain VIGRE fellowship support either as part of a recruiting package or through a competitive proposal process. Competitive proposals must discuss an intellectual, research-oriented, goal to be achieved during the period of support and are generally expected to have an integration component as well. Typical intellectual goals include: (i) study for qualifying exams; (ii) pursue a reading course or independent study with a faculty member; (iii) prepare for the written and oral comprehensive exam; (iv) organize a research seminar; (v) undertake a research project with a faculty member; (vi) prepare a paper for publication; (vii) present a paper or poster at a conference; (viii) completion of the dissertation. Activities that would add weight to a proposal include planned seminar talks and conference participation.

Integration activities might include any of the following: (i) organize (jointly with a faculty member) a research tutorial group (RTG) project; (ii) run a summer study session for the qualifying exams; (iii) act as "super-TA" for a graduate course or for Math 323, the critical transition to the mathematics major; (iii) act as a senior graduate student mentor in the Integration Workshop, or the Professional Skills seminar; (iv) work with a faculty member and an undergraduate on an undergraduate research project; (v) organize the undergraduate research seminar; (vi) participate in the High School Workshops program; (vii) visit, with a faculty member and an undergraduate, a local high school class; (viii) participate in Native American outreach activities, such as the Apache Summer Math Camp.

VIGRE funding provided on average slightly over 14 graduate fellowships per semester and about 15 summer fellowships per year. There is not sufficient space here to detail all the intellectual goals achieved and integration activities undertaken, but a comprehensive accounting, obtained from students' post-fellowship reports, is available. We feel that the number and range of activities undertaken is quite impressive and that students in the program during the period of VIGRE support obtained deeper and broader mathematical training as well as improved professional development.

VIGRE funding supported about 15 travel proposals per year. These were typically for conferences, both domestic and international, closely tied to the dissertation research of the recipient or for the joint annual meeting in January. Cases where students were presenting their own research were a priority. In many instances students were able to combine VIGRE funding with other sources (conference organizers, faculty in other departments, etc), leveraging the effectiveness of the VIGRE grant.

2.1.2 Program Statistics

We now turn to an analysis of the impact of VIGRE on our two graduate programs as a whole, comparing the pre-VIGRE period with the period of VIGRE support, 1999-2004. For this comparison, we feel that the appropriate time-frame for the earlier period is 1995-1999, the four-year period immediately preceding the VIGRE award. There were significant changes in admissions policies and program size for both the Mathematics Graduate Program and the Program in Applied Mathematics in 1995 (for example, both programs stopped accepting in most cases students they could not financially support), and so data for 1994 would skew the picture. (However that data for the full five-year period preceding the award is included in the appendix.)

Degree production The overall size of our graduate PhD programs declined somewhat between the two periods, averaging 91.3 students in the earlier period and 81.2 in the later period. The number of PhD students admitted was roughly constant in the two periods at an average of about 18.7 students per year.

The number of students leaving with no degree dropped from an average of about 4.5 per year to about 4.0 a year, i.e., an annual failure rate of only 5%. The number of PhD students leaving with a terminal Master's degree fell from an average of 5.7 per year to an average of 4.2 per year. (We remark that in the Mathematics program, a terminal Master's degree requires a thesis, and in the Applied program it requires a Master's level pass on the qualifying exams; in both programs the award of a Masters degree is far from automatic.) Most importantly, PhD production held essentially constant at about 10 degrees per year. We note by comparison that nationally, the number of PhDs awarded by Group I and II departments dropped by almost a third between 1997 and 2002. Moreover, the PhDs produced during the VIGRE period benefited form the extensive research and professional development opportunities afforded by VIGRE support. See the vignettes section for some examples. Overall, the picture is one of effective recruiting from a decreasing national pool, a steady success rate aided by VIGRE fellowship funding, and better mentoring leading to fewer total drop outs and fewer terminal Master's degrees.

Citizens, women, and minorities The proportion of US citizens and minorities in our incoming PhD classes held steady at approximately 71% and 31% respectively. (Here and elsewhere we abbreviate "citizens, nationals, and permanent residents" to "citizens.") The proportion of minorities among US citizens in our incoming PhD classes almost doubled, from 11.3% to 20.9%.

Among PhD degree recipients, the proportion of citizens decreased from 72.5% to 57.1%. Although this is a decline, the ending number is above the corresponding national average of 51.8% for the period '99-00 to '01-'02 (latest available, [Med04, p. 773]). Keeping in mind that there is a time lag of five to six years between admission and degree, the students graduating in the later period entered our programs between 1993 and 1999. Two factors may explain the lower percentage of citizens among graduates in the later period: the recruitment period was a time of great economic growth and many potential PhD students went directly from undergraduate degrees into the job market; and prior to 1995 we admitted some students without financial support; most of whom were eligible to work in the US, i.e., citizens and permanent residents. Looking ahead, we expect that the percentage of citizens among our graduates will rise in line with their rate of admission, which as noted above is about 70%.

The proportion of women among PhD recipients increased from 22.5% to 28.6%. (We note that among PhD programs of roughly our size, the top three percentages of PhDs awarded to females during 1995-2003 are 41%, 26%, and 25% [Jac04, p. 779].) The proportion of minorities among citizen degree recipients increased from 6.9% to 10.7%. (Over the last 10 years, approximately 4.8% of the PhDs awarded to US citizens and permanent residents have gone to minorities [Med04, p. 773].) Thus our record of producing female and minority PhDs is already among the best and we hope to see further improvements: we are only now beginning to see the graduation of students who have benefited from VIGRE during their full program of study and so we expect to see further improvements in these numbers, in line with the numbers for incoming students. Overall the picture is one of steady recruitment of citizens and women—in keeping with, or better than, national norms—along with improvements in minority recruiting, and improved graduation rates among women and minorities.

Placement The percentage of our PhD graduates obtaining academic or technical employment increased from 82.5% in the earlier period to 85.7% in the VIGRE period with a corresponding decrease from 17.5% to 14.3% in unemployment or temporary positions at the University of Arizona. This provides some evidence that our students are becoming more attractive to employers.

Among the employed the balance shifted from 73% academic, 27% industrial in the earlier period to 64% academic, 36% industrial in the VIGRE period, in line with VIGRE goals of broadening career prospects.

2.1.3 Vignettes

Sarah Frey Sarah Frey entered the Program in Applied Mathematics in Fall 2000 and, as one of the Program's top recruits that year, received a VIGRE fellowship in her first year, which was subsequently followed by a VIGRE fellowship in her fourth year. During her four years in the Program she has pursued interdisciplinary research on planetary stability under the joint supervision of Professor Michael Tabor (Applied Mathematics) and Professor Rick Greenberg (Lunar and Planetary Sciences). This work has already resulted in two publications with several more anticipated. In addition to a full slate of course

work Sarah taught several entry level mathematics courses and acted as a super-TA in one of the Applied Mathematics core courses. Sarah was strongly committed to outreach and, for 2001-2002, had an NSF GK-12 Fellowship working on curriculum development at a local Native American high school. Her professional training was enhanced by two summer internships at Los Alamos National Laboratory working on problems in computational fluid dynamics. During her fourth year she was offered a tenure-track assistant professor position in the Mathematics Department at Cal State Hayward. Given her very strong commitment to undergraduate education and outreach she decided to accept the offer and has just started her new job this semester.

Jack Hoppin Jack Hoppin joined the Program in Applied Mathematics in Fall 1998. During his time in the Program he received 6 semesters of VIGRE Fellowship support, four semesters of TA support, and additional semesters of RA support. He joined the University of Arizona's renowned medical imaging group under Regents Professor Harrison Barrett which has been the source of many interdisciplinary dissertation projects for Applied Mathematics students over the years. During his first year in the Program, he had considerably difficulty making the transition to some of the advanced graduate level courses and required extra faculty and senior student mentoring. Once over this hurdled he blossomed, and by the time of his graduation in 2003 he had received the Michael B. Merickel Best Student Paper Award at the 2003 Medical Imaging conference in San Diego, and was awarded a prestigious Humboldt fellowship for a post-doctoral position in Germany.

Guadelupe Lozano Guadelupe Lozano entered the Mathematics PhD program in 1996 and graduated in Summer 2004. Three years of VIGRE support enabled her to simultaneously pursue her primary research interest in integrable systems theory as well as a strong minor program in Mathematics Education. Her integration activities spanned the full range: helping to organize the Math Education research seminar, helping develop new courses for future teachers, organizing an RTG, serving as a super-TA, and being involved in outreach to the local high schools. She recently accepted a post-doctoral position at the University of Michigan where she is collaborating with faculty on extensions of her dissertation work in the area of Poisson Geometry as well as directing Michigan's Undergraduate pre-calculus program.

Tyler McMillen Tyler McMillen joined the Program in Applied Mathematics in Fall 1997. During his time in the Program he received a total of six semesters of VIGRE Fellowship support complemented by TA support in the other semesters (and was on a leave of absence for one semester for personal reasons). His vertical integration activities included involvement in high school workshops, super-TA work, and running review sessions for fellow students preparing for the qualifying exams. His dissertation research on "Perversion and whips: static and dynamic problems of elastic filaments" excited much popular interest, was cited in many professional and popular journals, and was featured on National Public Radio. By graduation this work resulted in three publications in major journals. Tyler graduated in 2003 and was awarded an NSF Post-doctoral Fellowship to work with Philip Holmes at Princeton.

Christopher Rasmussen Chris Rasmussen entered the Mathematics PhD program in 1998 and graduated in Spring 2004. He benefited from three years of VIGRE support, during which time he had a very strong research focus in arithmetical algebraic geometry. He gave a huge number of talks (averaging 3 a semester) attended several conferences, including two where he lectured, and organized a graduate student algebraic geometry seminar. His outreach activities focussed on high school AP calculus class visits. Chris accepted a VIGRE post-doctoral fellowship at Rice University in Fall 2004.

Patrick Shipman Patrick Shipman entered the Mathematics PhD program in 1999 and graduated in Spring 2004. He also benefited from about 3 years of VIGRE support, with a strong research emphasis on applied partial differential equations. He published one paper, gave three talks at conferences, attended 5 other conferences, and gave several talks in local seminars. His integration activities included participating in two Saturday workshops for high school students, organizing the undergraduate research seminar, and working twice as a super-TA in advanced courses. Patrick wrote his dissertation with Alan Newell and accepted a post-doctoral position at the Max Planck Institute in Leipzig.

There are many more interesting success stories among our current graduate students which may be discussed during a site visit.

2.2 Undergraduate Program

2.2.1 Statistics

Our previous VIGRE grant supported undergraduate students directly via fellowships for research projects and indirectly (with matching funds) for teaching projects; there were also funds for travel. The mechanics of these programs are discussed in Section 4.2, and here we will show that they have had a positive impact on both the numbers of graduates and their preparedness for careers in the mathematical sciences.

The number of undergraduate mathematics majors has increased (steadily except for a dip last year) from 282 in '98-'99 to 339 in '03-'04. The number of degrees awarded has been variable, with a peak of 59 in '02-'03 and an average of about 50 per year over the last five years. The percentage of degree recipients who have engaged in an independent research, teaching, or internship experience has increased substantially from about 25% in '98-'99 to over 53% in '03-'04, well above the national average. Based on a possibly non-representative sample, the percentage of degree recipients going on to graduate school increased from 18.7% in '96-'99 to 23.7% in '99-'04. (Our evaluation plan calls for much better data collection for statistics such as this one.)

The percentage of our undergraduate degrees going to women has held steady at about 40% over the last decade; this is roughly the national average. The percentage of degrees going to underrepresented minorities went from an average of 14.8% in the 1994-1999 period to 16.8% in the 1999-2004 period. The number of underrepresented minority mathematics majors is currently 20%, which compares favorably with the overall University of Arizona minority undergraduate student body percentage of 18%.

2.2.2 Additional facts

There is growing evidence that our undergraduates are improving in quality and professionalism. For Fall 2004 the first semester, upper level, courses in abstract algebra (Math 415a) and advanced calculus (Math 425a) are both oversubscribed, with enrollments of 35 and 44 students respectively. These very challenging courses are generally taken by students thinking of going on to graduate school in some mathematics-based field. At the professional development level, many of our undergraduates have undertaken research projects on both disciplinary and interdisciplinary problem, and in modalities ranging from traditional individual faculty supervision to vertically integrated teams. Furthermore, 16 of our students have internships for the summer of 2004 in companies and national laboratories.

2.2.3 Vignettes

Nathaniel Blair-Stahn. Nathan graduated with a B.S. degree in mathematics in December 2002. He was a UTA in a calculus class with duties that included holding weekly review sessions. He also held a URA, under the tutelage of post-doctoral associate Dr. Kelly Wieand, working on a project involving random matrices. During the summers of 2001 and 2002, he held internships at Sandia National Laboratory, where he worked on cryptography. He is currently in his second year of graduate study at the University of Washington.

David Brown. David graduated with a B.S degree in mathematics in December 2003. He worked on research projects with Professors Madden and McCallum (both in Mathematics), and with Professor van Klock (Physics). He spent the Spring semester, 2002 at the Budapest program. In the summer of 2003, he participated in a REU project at the University of Notre Dame, where he worked in combinatorics. He spent a year as a CAATS fellow, an NSF funded project, at Ha:San Native American preparatory school. David began graduate school at University of California, Berkeley in Fall 2004.

2.3 Post-doctoral Program

The post-doctoral aspect of our VIGRE program is small in numbers, but an important component. To document its success, we describe the trajectories of four of our VIGRE post-doctoral fellows, complemented by one statistic: all six of our former VIGRE post-docs are currently employed in the mathematical sciences.

2.3.1 Patrick Hagerty

Patrick Hagerty joined the Applied Mathematics Program in Fall 2001. In addition to developing his own research program in control theory, he engaged in highly productive vertical and horizontal integration activities. In 2001/2002 he mentored a mathematics undergraduate on a mathematical physics problem (cluster growth) proposed by senior faculty member Tom Kennedy; and working with a team of faculty members from the College of Engineering he supervised a research project for two engineering students on satellite stability and control. In addition he taught two undergraduate mathematics courses, and co-organized our annual Los Alamos-Applied Mathematics research conference. (Patrick's wife was assigned a medical internship in Baltimore and after one year at Arizona he accepted a position at the NSA in Washington so that they could be together.)

2.3.2 Maurice Hasson

Maurice Hasson joined the Applied Mathematics Program in the Fall of 2001 as its second VIGRE post-doc after completing his PhD at Rutgers. Having made a bold mid-career change to become a professional mathematician he brought the intellectual and personal maturity of an older individual to this position. This was quickly evidenced by the independence and productivity of his research program and sophistication of his pedagogy. Over the past three years he has taught a broad range of undergraduate mathematics courses and two graduate level course on wavelets and harmonic analysis which are one of his areas of expertise. In addition to these courses he has run a series of intense and comprehensive review sessions for Applied Mathematics students preparing for their qualifying exams. Since being here he has produced and submitted seven manuscripts of which four are independent work and three have already been accepted for publication. He is mentored by Professor Michael Tabor (Head of Applied Mathematics) with whom he has co-authored two papers.

2.3.3 Mark Inlow

Mark Inlow received a PhD in Statistics in 2001 from Texas A&M University and joined the Mathematics Department as a VIGRE post-doc in Fall 2001. He was a notable example of "horizontal integration," i.e., integration across fields. He had numerous interactions with the Arizona Cancer Center through their bioinformatics colloquium and he taught Advanced Biostatistics in the School of Public Health. In the Mathematics Department, he organized the statistics colloquium and actively advanced his own research program, publishing three papers and beginning another which he presented at a national conference. He left Arizona in 2003 to take up a tenure-track position at Rose-Hulman Institute of Technology.

2.3.4 Romyar Sharifi

Romyar Sharifi received his doctorate in Mathematics from the University of Chicago in 1999 and joined the University of Arizona Mathematics Department in January 2000 after a semester at MSRI. While here he initiated a powerful research program centered around the structure of certain infinite Galois groups, Iwasawa theory, and modular forms, which resulted in several papers, one of them joint with Professor William McCallum. He supervised an RTG in 2000-2001 on number theory. In Spring of 2001 he was awarded an NSF post-doctoral fellowship to work with Barry Mazur at Harvard and so left Arizona after three semesters. He is currently a tenure-track assistant professor at McMaster University and he is being nominated for a Canada Research Chair.

2.4 Faculty Involvement

Faculty involvement in our VIGRE program has been widespread. Defining "involved" as supervising a URA, UTA, or RTG project, a Master's or PhD thesis, teaching a core course, or participating in the Integration Workshop (all activities that involve extensive student contact), at least 50 of our 64 tenured or tenure track faculty members have been involved with VIGRE in the 1999-2004 time frame. This represents a participation rate of over 78%. In fact, several of the non-participating faculty members were hired recently and so one may expect even higher rates of involvement in the next VIGRE project.

3 Outcome of Curriculum Review

3.1 Graduate Programs

Both the Mathematics and Applied Mathematics graduate programs consider curriculum review to be a continuousprocess and while some aspects of the two programs' curricular structures are similar, others take advantage of their unique individual strengths. The guiding principles enunciated in Section 4.1.1 apply equally to both programs and govern the approach to curriculum development undertaken by the programs, a process that involves broad representation of research groups, graduate students, and, in the case of the Applied Mathematics program, faculty from other disciplines.

As a starting point in our thinking about this proposal, we conducted an in-depth analysis of our students' trajectories over the last 10 years, focussing on the reasons why students might not complete a degree, or take overly long to do so. Not surprisingly, we found that the main stumbling points were the qualifying exams and, to a much lesser extent, the transition from course work to preparation for the comprehensive oral exam and the thesis. Among those students not getting past the qualifying exams, some were simply weak students and thus represent missteps in recruiting; our analysis indicates that others might well have done better with more structured advising and pro-active placement into undergraduate courses needed to fill gaps in their backgrounds. Thus we have undertaken new initiatives in recruiting, described in Section 5.1 and in the first two years of the program, described below and in Sections 4.1.2-4.1.4. That the transition to the thesis is much less of a problem indicates that our post-core advising mechanisms are working well, although here too we have some improvements to propose.

Over the past three years both graduate programs have introduced significant changes to the curricular structure and assessment mechanisms used in the first two years of study. The transitional phase into the first year of graduate school is supported through two components developed under the previous VIGRE grant, namely the "Integration Workshop" in the Mathematics program and a "Skills and professional development seminar" in the Applied Mathematics program. These components are complemented by the successful system of Research Tutorial Groups used by both graduate programs, which provide a mechanism to start students on supervised research experiences as soon as possible. As originally formulated, RTGs were successful in helping students begin the transition from course work to research, but a careful assessment indicated that their scheduling (Spring of the first year) put an undue burden on the typical student preparing for qualifying exams. Accordingly, they were shifted, for both graduate programs, to the third semester and implemented in an expanded form. In addition, the RTG system has been complemented by a system of core-course related term papers in the first year. These provide a first transition point to independent student effort under careful faculty guidance. All these recent changes implement our principle of introducing authentic mathematical practice as early as possible, and of maximizing the opportunities for the students to demonstrate their individual strengths outside the framework of traditional course-work and exams.

In the past year both graduate programs have undertaken sweeping reviews of their "post-core" requirements and set new standards for satisfactory academic progress. In the case of the Mathematics graduate program these changes are focussed on aiding with, and realistically advising students about, the transition from the qualifying exam stage to the dissertation stage by way of the Comprehensive exam, and on making the professional development requirements for the PhD more flexible and "authentic." Specifically, the Mathematics Program's new satisfactory progress guidelines institute checkpoints at which the graduate committee may require, based on all available evidence (qualifying exams, Research Tutorial Groups, term papers, and course work) that certain students complete a Master's thesis before continuing to the PhD. Our experience is that rather than adding time to the degree, the MS thesis forces students to confront various issues, such as time management, significant writing, and efficient mastery of new material, in a more manageable context that will help them gain the skills and confidence necessary to attack the Comprehensive exam and PhD dissertation research. Regarding professional development requirements, the Mathematics Program is moving to a system of projects and other activities, reflective of things mathematicians really do as part of their professional lives, as the preferred way to demonstrate competencies in computing, communications, and foreign languages. Thus rather than taking a timed exam in a programming language, students will carry out a computing project related to their research using the appropriate language or package. Moreover, students are free to choose two of the three requirements, with the proviso that faculty advisors are free to require some, or even all three, of the professional development requirements for their

students. Thus students working in areas, such as arithmetical algebraic geometry, where reading French is crucial will have that skill while others, such as those working in certain areas of applications oriented mathematics, will focus more on computing. The goal is that students amass, naturally and in the course of their PhD programs, the set of skills most relevant to their careers.

In the case of the Applied Mathematics Program the post-core changes have also been quite substantial. One of the most notable, and original, features of the Program's course requirements was that of five courses outside mathematics. After careful analysis of the historical data, this is now being changed to a new "3+3+3" requirement in which students are required to take, beyond the core courses, at least three courses outside mathematics, three approved advanced-level courses in mathematics, and three others of their choosing either in or out of mathematics. This change gives the students greater flexibility in their programs of study, while still preserving the interdisciplinary principles of the Program by ensuring that neither their advanced mathematical training nor their interdisciplinary education are compromised at the expense of the other. In order to monitor satisfactory academic and professional progress the Program is now adding additional components to the oral comprehensive exam: at the time of the exam students must also present (i) a "road map" of their post-exam plan of study and professional development; (ii) an up-to-date and professional looking resumé; (iii) a professional web-page. Students who have not completed their oral comprehensive exams by the end of their third year will now be required to undergo a third year review process with a small faculty committee with the goal of helping them find an appropriate research area, and to discuss any corrective strategies that may be deemed necessary to ensure timely progress. Finally, the old foreign language exam requirement has been completely dropped in favor of the optional alternative of proposal writing. Now, all students will be required to submit a draft proposal for an NSF (or other national funding entity) post-doctoral fellowship before their final dissertation defense.

Other changes of the past few years have included the introduction of "super-TA" positions in which senior graduate students who assist in various upper level undergraduate and graduate courses—especially the core courses for first-year graduate students. (See Sections 4.1 and 4.2.5.) Several important course development projects have also been implemented: The Applied Mathematics core courses, especially Methods of Applied Mathematics (Math 583) and Numerical Analysis (Math 575), have been refined and restructured; and the advanced graduate courses in Banach and Hilbert Spaces (Math 528) and Ordinary Differential Equations (Math 554) have been revitalized, and a new graduate introduction to Mathematical Physics has been added.

3.2 Undergraduate Program

Our undergraduate program also operates a system of continuous curriculum review, overseen by the Associate Head for Undergraduate Affairs and by the Undergraduate Committee, which is governed by the guiding principles of our undergraduate program (enumerated in Section 4.2.1). Again, the committee has broad representation of regular faculty, post-docs, adjunct faculty, graduate students, and departmental administration and major initiatives are reviewed by the entire voting faculty.

We have instituted a special course, Math 250ab, for incoming freshman with a score of 4 or 5 on the AB Advanced calculus placement exam. This course gives students an integrated approach to integral calculus and differential equations, and in the hands of master teacher David Lomen has proved an effective means of recruitment into the major.

More recently, we have been looking at ways to improve the success rate in our transition to the major course, Math 323, Formal Mathematical Reasoning and Writing. We have involved graduate students in the course as super-TAs and have initiated a dialog on incorporating results from research in mathematics education into our classroom practice. We are also planning a one-unit course for math majors as an aid in the transition to the major and the profession. See 4.2.5 for more on these initiatives.

Several new upper-division courses have been developed to broaden the math major and introduce students to career directions other than graduate school: at the 300-level, Math 381, the Mathematics of Finance, was introduced in Spring 2001, and Math 396C, Undergraduate Research Seminar, a one-unit course designed to ease students into our undergraduate research program, was introduced in Spring 2004 (and ran informally for three years before that). At the 400 level, we started Math 408, Harmonic Analysis, in Fall 2002, in order to give students an experience where both analysis and algebra were combined to study one phenomenon, and Math 445, Introduction to Cryptography, was introduced in Spring 2004. We are also starting to develop capstone courses; these are described more fully in Section 4.2.6.

4 Proposed Project

In the following sections we give detailed descriptions of our graduate, undergraduate, post-doctoral, and summer training programs. These programs do not operate in isolation and, as described below, an infrastructure of vertical and horizontal integration mechanisms, set in a vibrant and collegial academic environment, weave them all together into a cohesive and mutually supportive community of students, researchers, and teachers. The new summer program will be an integral part of this environment: contributing to and reflecting our inclusive culture of research and education in the mathematical sciences. The project activities described below are a combination of new initiatives and the successful components developed under our past VIGRE grant. The latter include the system of competitive proposal writing, the Integration Workshop, the Skills seminar, first-year term papers and third-semester research projects, and new or revised undergraduate courses. These recent components, and the new programmatic activities, will be subject to careful evaluation and assessment as the project evolves.

4.1 The Graduate Programs

4.1.1 Goals and guiding principles

We describe, in approximately chronological order, the various transition points experienced by our graduate students and the mechanisms for guiding the students through them. The described programmatic design, for both graduate programs, is the result of years of careful experimentation and evaluation based on the following principles:

- Study plans must be designed flexibly to accommodate diversity of educational backgrounds.
- Students should be able to demonstrate their abilities and research potential outside traditional coursework and examination metrics.
- Verbal and written communications skills must be developed at all stages of students' graduate careers.
- Students need to be exposed to a broad range of authentic research and teaching experiences as early as possible in both vertically and horizontally integrated settings.
- Professionally broadening experiences, ranging from outreach to internships, are valuable preparation for careers in the mathematical sciences

Although there are many differences in the curricular requirements of the Mathematics and Applied Mathematics programs, the overall goal of giving each student the very best opportunities to maximize his or her potential is the same.

4.1.2 The transition from undergraduate to graduate school

Our goal is to effect a smooth integration of new students into their respective graduate programs, and to begin their preparation for the study of mathematics at the highest level. For both graduate programs the process begins at the Spring Recruitment Workshop (see Section 5.1 for more details) and continues over the summer prior to the first semester. During this period faculty are in communication with the entering classes about the expectations of graduate school, background study and summer reading suggestions, and teams of current graduate students are available to advise students about practical and logistical issues.

In August, just prior to the beginning of classes, the Mathematics Department holds a week-long, "Integration Workshop" to initiate incoming graduate students into the Mathematics program. The goals of this workshop, which was started two years ago and modeled on the long standing Arizona Winter School, are to: (i) prepare students for core classes by reviewing undergraduate material from a graduate point of view; (ii) introduce students to the process by which complex mathematical problems are broken down and tackled by the appropriate mathematical tools; (iii) establish an *esprit de corps* among the students, and acquaint them with key faculty and current graduate students. The workshop is led by the Associate Head for the Graduate Program of the Mathematics Department assisted by a team of faculty and a selected group senior graduate students for whom it is a valuable professional experience in mentoring and instruction. Design

elements borrowed from the Arizona Winter School, such as a very intense schedule and significant projects presented at the end of the workshop, have transferred well to this context. The integration workshop is followed up with a series of interviews with each participant to design a first-year program of study that will maximize their probability of success in the first year.

In Applied Mathematics the integration transition is accomplished through a semester-long "Professional Skills and Development seminar" in the Fall which takes advantage of the Program's unique experimental facilities. It currently consists of: (i) carefully designed, laboratory-based, team projects to provide the first taste of training in the scientific methodologies so necessary for applied mathematics; (ii) communication skills training in which each project is presented in a Power-Point-based talk, and as journal-style article prepared in LaTeX; (iii) additional activities that include a number of case-study based scientific ethics discussions relevant to interdisciplinary research. This course is taught by the Head of Applied Mathematics and a highly experienced senior laboratory manager. (More details of our unique laboratory facilities can be found in Section 4.6.1.) Graduate students taking the course are encouraged to consider ways in which their projects could be used to demonstrate the applications of mathematics at the undergraduate and high-school levels, and the experimental modules used in this course will, with appropriate modification, be used in our summer program.

4.1.3 Laying the foundations: core courses and first research and teaching experiences

This all-important foundational phase, which typically covers the first three to four semesters in each graduate program, is critical for both preparing the students for advanced research and teaching, and assessing their ability to complete a PhD. During this phase students in both programs take a rigorous and integrated set of core courses. Subject to a careful analysis of their background, students have the option to complete their core studies in one or two years. As they undertake these studies, students are expected to develop independent and efficient work habits in their coursework. There is careful coordination among the core course instructors, and regular contact between them and the appropriate program head who monitors each student's overall progress. Senior graduate students play the role of "super-TAs" in the core courses by providing review and problem sessions for the entering class.

During their second semester students in both graduate programs undertake a substantial term paper project connected with one of their core courses. The term paper provides the students with the opportunity for creative exploration of an advanced topic, and provides a first indicator of their ability to function independently. Students are required to submit journal style reports in LaTeX of their project, and present their papers at the end of the semester in term paper mini-conferences. The introduction of term papers into the first-year syllabus was a consequence of a careful assessment of a previous requirement of an independent first-year research project: in many cases the students had insufficient technical background for the project to be an authentic research experience. By tying the term paper to the core courses and the core instructors a much greater uniformity of level, and faculty oversight, has been achieved.

With two semesters of rigorous core courses behind them, the students are equipped to undertake a first, and meaningful, independent research project in their third semester - this is our Research Tutorial Group (RTG) system. Each project, which can be individual or team based, is vetted by the appropriate program head at the beginning of the semester and is presented at the end of the semester at research conferences held by each program. Opportunities for possible projects are presented to students, in both programs, during the course of their first year through a system of evolving (and separately organized) mini-courses, case-study talks by faculty, and "data-blitz" style sessions that enable a larger number of faculty to briefly introduce themselves and their research programs to the students. Both the third semester research project and second semester term paper play a significant role in the assessment of each student's progress.

Students have their first teaching experiences in either their first or second year. This takes the form of teaching a section of an entry level course (usually college algebra or trigonometry), and students are expected to take full responsibility for their course section. Each TA is under the supervision of an experienced course coordinator, and is required to attend a weekly course meeting. Moreover, all first-time TAs participate in a year-long, one-unit Professional Development Workshop in Teaching Mathematics. These mentoring and support mechanisms enable the students to learn basic teaching skills in a carefully supervised environment.

4.1.4 Qualifying exams, performance assessment, and the transition to advanced course-work

In both graduate programs, students have up to two years and two attempts to pass their qualifying exams at a satisfactory level. Students who complete their core course requirements in their first year usually take their exams during the summer at the end of that year, while students on a two-year core attempt to complete their exam requirements by the end of their second summer. During the summer session prior to the exam, advanced graduate students and post-doctoral fellows run review sessions and support teams to help students in their respective graduate programs prepare for the qualifying exams. For the student mentors this activity can be part of their VIGRE fellowship activities.

While the qualifying exams are a traditional and useful measure of technical expertise, they need to be combined with other metrics to assess a student's ability to successfully complete a PhD. It is here that their performance in the term paper and, for students on a two-year core or retaking the exam, the third semester research project play a very valuable role. Implementation of this multiple-component assessment process has resulted in more satisfactory students outcomes, and will help minimize attrition in the future.

The consequences of the exam results are essentially the same for students in both graduate programs. Based on the results, students plan their next year of study in careful consultation with their program head. Students with PhD passes have considerable flexibility in selecting advanced course work, while those needing to retake the exams design their programs of study with an eye to the appropriate remedial actions.

In mapping out a comprehensive program of course work, breadth of education is an important component for both graduate programs. Students in the Mathematics program are required to take two courses outside mathematics, and students in the Applied Mathematics program are required to take a balance of courses outside mathematics and advanced mathematics courses.

4.1.5 The intermediate phase: finding a research area, developing an individual teaching style, and the Comprehensive exams

This phase, which typically begins after successful completion of the qualifying exams, marks the beginning of a much more independent effort on the part of the students as they make the transition from being a consumer of knowledge to a producer of (their own) ideas, and an effective teacher. This critical transition has many components: acquiring advanced technical knowledge, in-depth exploration of research opportunities, and developing the ability to strategize and prioritize a research project—all set in a collegial climate in which research is discussed enthusiastically and openly without regard to academic rank. The expected outcome of this phase is the identification of an advisor and likely dissertation topic, and preparation for the comprehensive exam.

An important mechanism for fostering independence and developing a research strategy at this stage is the writing of competitive grant proposals for VIGRE funding. All students seeking VIGRE fellowship support after completing their qualifying exam requirements must write a research proposal describing the specific goals they intend to achieve with the funding in the areas of research, teaching, and outreach; and the proposal must contain at least one vertical integration component. Mentorship is provided by a review process in which the first draft of the proposal is discussed in an interview with the VIGRE management committee—which makes suggestions for changes—leading to a final version of the proposal which must be accompanied by a letter of support from the faculty member most involved with the proposed activities. It should be noted that not all VIGRE proposals are funded—an experience which constitutes an important lesson in the realities of academic life.

The exploration of possible research areas is achieved in a variety of ways including: pursuing outgrowths of the required third-semester research project; arranging independent study projects with faculty members; taking advanced and special topics courses (for students in the Applied Mathematics Program this often includes substantial cross-disciplinary course work in areas outside mathematics); and attending and participating in working seminars of the many research groups on a regular basis.

An important component of a student's transition to an independent thinker is the University-mandated comprehensive exam. This stage requires the student to have identified an official advisor, and to have assembled his or her dissertation committee. The essence of the exam is for the student to demonstrate the necessary intellectual maturity to undertake dissertation-level research through convincing written and oral presentations of a research topic, and the ability to respond to intense questioning on and around it. Furthermore, both graduate programs have introduced the additional requirement that students must

present a multi-year "road-map" that describes their plan of study and pursuit of professional development opportunities to be undertaken after the exam. As part of our process of continuous curriculum review we note that both graduate programs are now introducing additional mentoring and monitoring components appropriate to this phase to ensure timely progress in these intermediate years. These are discussed in more detail in Section 3.

In the case of research in applied mathematics additional components are required to achieve greater research maturity: namely exposure to the scientific methodologies associated with the analysis and interpretation of data, and the ability to master the fundamentals of other scientific disciplines. The Applied Mathematics Laboratory provides a unique and advanced experience in empirical methodologies through projects involving the synthesis of sophisticated mathematical modeling with non-trivial laboratory experimentation and data analysis; while the degree requirement of taking courses in other fields—usually tied directly to their interdisciplinary research goals—significantly broadens the students' knowledge base.

The teaching goal of this phase is for the students to develop into more confident instructors with a flexible approach to different courses and students with diverse backgrounds, and to develop a competitive portfolio of teaching experiences. VIGRE support provides students with the opportunity for additional instructional experiences including: supervising an undergraduate research project in a vertically integrated team involving faculty and postdocs; being a super-TA in a graduate course; and organizing or participating in outreach projects involving high school and minority students.

4.1.6 The dissertation phase and professional development

Successful completion of the comprehensive exam marks the beginning of the student's dissertation phase. At this stage the primary mentoring is provided by the student's advisor, but the process of applying for VIGRE fellowships continues to provide an important mechanism for students and their advisors to assess progress and discuss career options.

The dissertation phase is often an appropriate time for the student to engage in various professional development opportunities, such as participating in a conference or workshop, or undertaking a summer internship to explore career options outside academia. There is already a long-standing tradition among students in the Applied Mathematics Program of undertaking summer internships at Los Alamos National Laboratory, and a growing interest in such opportunities among the Mathematics students. Mac Hyman, a senior figure at Los Alamos and an Adjunct Professor in the Mathematics Department, frequently visits the department and advertises research opportunities at the laboratory. Abbie Warrick and Evi Dube (both graduates of the Mathematics undergraduate program and the Applied Mathematics graduate program) from Lawrence Livermore, and Cheryl Beaver (a graduate of the Mathematics graduate Program) currently at Sandia, visit regularly to give talks to our graduate students about opportunities at their respective laboratories.

Attending workshops and conferences, and presenting research results at professional society meetings, is an important part of a graduate student's professional development. Students in both programs regularly participate in national and international meetings, and VIGRE funds are a valuable source of support to help them do this. We also encourage participation in programs sponsored by the various NSF national institutes in the mathematical sciences and, in particular, participation by local graduate students in the Arizona Winter School, a premier program for graduate students working in number theory and algebraic geometry near completion of their dissertations. Over the last five years the Applied Mathematics Program has organized an annual biomathematics workshop at which local students interested in this field participate.

One of the pleasures of academic life is to see graduate students blossom into enthusiastic and creative researchers and teachers with ideas and opinions of their own making, able to engage and challenge faculty and visitors in scientific and professional discussions, and able to inspire and mentor the younger students. These are the young men and women that our collegial academic environment aims to produce.

4.2 Undergraduate Program

4.2.1 Goals and guiding principles

Our undergraduate program also involves several phases and critical transition points supported by the appropriate mentoring mechanisms. The overall design of the undergraduate program and associated mentoring

structures is guided by the following principles:

- Recruitment into the major can begin as early as high school and continues through the Mathematics Department's "service" courses such as calculus, linear algebra, and differential equations.
- Particular attention is paid to the recruitment and needs of women and under-represented groups.
- The mathematics major opens the doors to many professions, as well as graduate study, and should provide options that prepare students for a variety of career trajectories.
- A well-designed undergraduate mathematics major should include research, teaching, and internship experiences.

Graduate students and postdocs, as part of their VIGRE integration activities, will often be key players in the teams that help undergraduates pass critical transition points. They are often less intimidating than faculty and can help build an excellent mentoring environment for undergraduates.

4.2.2 The Math Center

Most of our efforts in outreach to undergraduates are coordinated by the Department's Math Center which is led by Associate Head for Undergraduate Affairs, William Velez. The Math Center is the hub of activities for undergraduate mathematics majors with responsibility for coordinating recruitment, outreach, retention, and advising. Additionally, Velez continues our serious efforts to increase the diversity of the undergraduate program; and he also serves as industrial liaison for the Department with the goal of increasing the number of students applying for summer internships with industry and national laboratories.

The Department's commitment to the undergraduate mathematics program is demonstrated by its significant allocation of space and personnel to the Math Center: it now has a full time academic professional and a full time Administrative Assistant to coordinate the growing number of activities that have resulted from the increasing number of undergraduate mathematics majors and minors.

4.2.3 The transition from high school to university

Many high school students are unaware of the benefits of studying advanced mathematics, or of the career opportunities available to math majors. During our first VIGRE grant we started a program of sending three-person teams (faculty, graduate student, and undergraduate) to visit calculus classes in local high schools, with the goal of encouraging these students to enroll in more mathematics classes when they attend the university and to think about becoming mathematics majors. In parallel with this we now have a program in which graduate students organize and present after-school workshops for high school students. This activity is entirely organized by the graduate students, who contact local high schools and set up two- to four-hour long workshops. Several workshops are held each semester on a variety of mathematical topics. Both the high-school visits and outreach workshops will continue under the new VIGRE program.

It is well recognized that incoming undergraduates frequently have a difficult time making the transition to the university setting, especially when the first semester consists of courses in calculus, science, and computer science. Students often do not understand the change of pace that is about to occur in their university classes, and we have developed a variety of mechanisms (such as reading assignments and exercises) to help ready them for their first calculus course on campus. This approach builds on an advising program for minority calculus students run for many years by Associate Head Velez.

Further efforts to help minority students make the transition from high school will be aided by CEMELA (the Center for Mathematics Education of Latinos/as), a newly established, NSF-funded Center for Learning and Teaching (see Section 4.6.6 for more details).

4.2.4 Lower division course work

Once future mathematics majors arrive on campus they take a rigorous program of courses in calculus, differential equations, and linear algebra. We use enrollments in calculus classes as a mechanism for identifying potential mathematics majors. Furthermore, many of the minority students enrolled in calculus are interviewed and encouraged to take more advanced classes, and to consider becoming mathematics majors. The

result of these efforts has been that over the last three years the University of Arizona has had on average over 300 mathematics majors and currently 60 of these are minorities.

Building on this success, the teaching staff for the lower division courses, with the support of the Math Center, are participating more fully in recruiting students into the mathematics major. Towards this end the Department has developed a new protocol involving letters and emails to instructors and potential majors that is proving to be effective in encouraging students to become mathematics majors.

A successful VIGRE-related program that impacts on lower division course work is our Undergraduate Teaching Assistant (UTA) program, described more fully below. In this program, advanced undergraduates serve as teaching assistants in lower division math courses.

4.2.5 The transition to upper division work

During the first semester of the junior year students typically take Math 323, Formal Mathematical Reasoning and Writing, in which they learn how to master logical reasoning and to construct a proof. While our best students often bypass this course and move to upper division coursework directly, it is immensely challenging for our typical students, and is probably the single most important transition point for a successful mathematics major. We have two initiatives to improve the success rate in this course. First, beginning in Spring 2004, we assigned a special graduate student teaching assistant (a "super-TA") to each section of this course, in order to give students an additional resource to learn the material. Second, we have initiated a dialog between the instructors, the super-TA, and the Associate Head for Undergraduate Affairs around recent research in math education relevant to this course. Specifically, recently published research on students' understanding of the concept of mathematical definition (see [EW04]) has implications for classroom practice. It is currently rare for research in math education to motivate changes in classroom practice (as opposed to the other way around) and we hope to see this integration of education research and education spread to other courses. It is worth noting that the topic of learning at the critical transition to upper-level course work has been the topic of two recent dissertations by students in our Math Education Program ([Smi02] and [Jim04]).

4.2.6 Upper division course work

Once they have mastered the foundational lower-division material, math majors at Arizona have an exceptionally rich choice of further topics to study. Our undergraduate program currently has six tracks (Comprehensive; Applied; Probability and Statistics; Computer Science; Economics and Finance; and Education) with another (Biomathematics) under development. We have also recently introduced several new upper division courses, including Mathematics of Finance, an Undergraduate Research Seminar, Harmonic Analysis, and Cryptography.

We continue to add capstone courses to our major. One example is the redesigned Mathematical Modeling course, which received financial support from the University for upgrading classroom technology. The new version of this course, which first ran in the Spring of 2005, involves VIGRE-supported graduate students who begin the course by giving presentations describing various modeling projects (which could include an Applied Mathematics Laboratory component), and then help mentor the undergraduates as they work on their chosen projects during the semester. The course emphasizes communication skills: students have significant writing assignments, and prepare posters about their projects. The teams that put together the best posters are sent to the undergraduate poster session of the next Joint Mathematics Meeting, and also present their work in the undergraduate research seminar described below. Further topics for the modeling course will be developed by teams of VIGRE-supported undergraduates and graduate students during the summer program described in Section 4.4. Other candidates for capstone courses include Cryptography and Synthesis of Mathematical Concepts, a course for future high school teachers which investigates high school mathematics from an advanced perspective.

4.2.7 Enrichment opportunities and the transition to professional careers in mathematics

In this all-important phase, students learn to consolidate what they have learned as undergraduates and put together various concepts in order to solve a problem, work on a research project, or present sophisticated mathematics in preparation for future careers as mathematical professionals. To support this transition, the

Department has for many years offered Undergraduate Teaching Assistantships and Undergraduate Research Assistantships, and the Math Center and faculty advisors also help students apply for summer internships. Participation in some or all of these activities gives a student a very well-rounded undergraduate experience involving the creation, application, and communication of mathematics.

Undergraduate research program. For undergraduates, the experience of trying to solve large, often poorly formulated problems is of tremendous value, and they have repeatedly told us how much difference this makes in their appreciation of mathematics. Furthermore, graduate students involved in mentoring undergraduate research projects gain valuable experience in communicating mathematical concepts and developing research strategies. And faculty also benefit—the enthusiasm of young students, and the "outside the box" questions that they often raise, re-invigorate our own investigations.

Undergraduate research opportunities are presented in our Undergraduate Research Seminar offered in the Spring semester. It is modeled on the expository phase of the graduate Research Tutorial Groups, in which faculty give a three- or four-week overview of selected topics. These faculty are then available to supervise research projects in the following semester. These projects can include both students supported by VIGRE and those supported by departmental funds (available for non-U.S. students).

The continued vitality of this activity is important for our undergraduate program and we are seeking ways to permanently fund it with other resources. As an additional mechanism in the transition to post-VIGRE funding, we plan to involve research groups in the generation of URA funding by encouraging (senior) faculty members in the group to apply for funding for undergraduate research in their research proposals. These modes of funding will be phased in over five years while the URA component of our VIGRE budget is phased out—with the goal of maintaining a steady state of funded undergraduate research activities.

Another new initiative related to undergraduate research is the Arizona Mathematics Undergraduate Conference (AMUC), started in 2004, at which students present their research and interact with other mathematics majors from around the state. AMUC is a yearly event which rotates among the three state universities of Arizona. The Conference also serves as a valuable recruiting tool for both of our graduate programs since it provides a venue for our faculty to identify and interact with some of the brightest students in the region. Participating students from the University of Arizona will be supported by the undergraduate travel budget of our VIGRE grant.

Undergraduate teaching assistants. Each semester the Department supports ten Undergraduate Teaching Assistants (UTAs) in a program managed by a teaching post-doc (see Section 4.6.10 for a brief description of this program). Each UTA works ten hours per week. Four hours are devoted to tutoring in the Department's tutoring program. Five hours are spent working with an assigned course instructor on a range of activities: some UTAs run extra problem sessions for the course; others help the instructors with special projects (e.g. developing software to help explain a topic in the course); and others may give a lecture, or part of a lecture, in the course that they have been assigned to. The work-load is completed with a weekly, one-hour, session run by the teaching post-doc for all the UTAs to discuss common concerns and the activities that they are involved in.

To our knowledge, there are no federal sources of funding for activities such as the UTA program. Therefore, as part of our post-VIGRE plan, we have begun the process of shifting funding for the UTA program from VIGRE to an endowed scholarship fund. We have established the Lusk scholarship (funded by a private donor) which is currently sufficient to fund a small number of UTAs and we hope to expand this over the next five years in order to fully fund ten UTAs per semester. Simultaneously, the UTA component of our VIGRE budget will be phased out over the lifetime of the grant.

Internships. Summer internships can be seen as huge reward for those students who are willing to work diligently on their studies. Internship leads have been developed by Associate Head Velez (acting as industrial liaison on our first VIGRE grant) and by Professor Juan Restrepo, through the internship program (AIMES) he originally developed at UCLA.

These internships are very competitive and handling the challenges of applying is an important step in a young student's professional development; guidance is provided by the Associate Head and Math Center staff.

For the summers of 2003 and 2004, about 30 mathematics majors obtained internships. For summer 2005, at least 35 majors obtained internships. These have been at national laboratories (Lawrence Livermore, Sandia), technical firms, (Raytheon, IBM), and REU sites at the university and around the country. We are also helping students to apply for semester-long programs abroad such as the programs in Moscow and in Budapest.

4.3 Post-doctoral Program

The overall goal of this important component of our VIGRE program is give the postdoctoral fellows a well-rounded training in the mathematical sciences and a broad-minded perspective of the discipline as a whole—reflective of the collaborative culture of Arizona— that will serve them well in their future careers. By the end of their fellowships they will have:

- A productive and innovative research program.
- A broad portfolio of teaching experiences including the possibility of team teaching with other departments.
- Experience of interdisciplinary interactions.
- Experience of vertical integration activities and mentoring students at different levels.
- Well-developed communication skills.

Post-doctoral fellows clearly play a vital role in a VIGRE program as agents of both vertical and horizontal integration, and as valuable contributors to our research programs. For example, Applied Mathematics VIGRE postdoc Maurice Hasson ran a remarkably effective series of tutoring and review sessions for the Applied Mathematics students preparing for their qualifying exams, and contributed significantly to the research programs of his faculty mentors, Juan Restrepo and Michael Tabor. Vertical integration successes included an RTG led by Mathematics post-doc Dennis Eichhorn in 2000 on number theory and partition theory which led to two Master's theses. Horizontal integration efforts included Mathematics post-doc Mark Inlow helping teach statistics courses in the College of Public Health, and Applied Mathematics post-doc Patrick Hagerty supervising a satellite control and stability project with students from Engineering and Planetary Sciences.

Our experience has been that despite their greater maturity, the post-doctoral fellows still need careful mentoring to ensure that they are properly integrated into the many activities of the Department and Program, and that they develop a balanced portfolio of research, teaching, and integration activities. The integration of the post-doctoral fellows begins at the recruitment phase, with research groups in the Mathematics department proposing candidates for recruitment, specifying the range of VIGRE activities they could be involved in, and identifying a specific faculty member (or members) to serve as primary mentors. The Applied Mathematics post-docs is recruited by a separate committee involving the participation of Program faculty in other departments where interdisciplinary opportunities are possible. Once here, the VIGRE post-doctoral fellows associated with the Mathematics Department are supervised by the head of Mathematics, and the Applied Mathematics post-docs by the head of that unit. This mentoring structure ensures that the post-docs benefit fully from the wide range of opportunities that are available to them here, and that they are quickly integrated into seminar programs and research activities.

Teaching activities are at the level of one course per semester and designed to ensure experience at a range of instructional levels; and a program of vertical integration activities is developed at the beginning of the fellowship. Communication skills are developed through the usual range of activities, and honed by the writing of a research proposal to an external funding agency in the second or third year of their appointment under the the guidance of a senior faculty member. Additional mentoring in professional development activities, especially those pertaining to teaching, will be provided by the head of the Teaching Post-Doc Program, Elias Toubassi.

Overall, our unique and integrative intellectual environment provides an ideal setting in which the post-doctoral fellows can flourish and move on to successful careers as professional mathematicians. Thus Romyar Sharifi went on to an NSF post-doc at Harvard and now has a tenure-track position at McMaster, Mark

Inlow has a tenure-track position at Rose-Hulman, Patrick Hagerty has a position at the NSA, and Maurice Hasson is at Bucknell.

4.4 Arizona Summer Program

The proposed summer program is a new and important component of our workforce enhancement efforts that will provide an extension of our teaching and training paradigm by establishing intellectual and cultural interfaces with regional schools. The over-arching goal is to develop the pipeline of talented students from diverse backgrounds and institutions into the mathematical sciences professions. This will be achieved by:

- Providing undergraduates with stimulating research and professional development experiences that will encourage them to attend graduate school.
- Demonstrating the unity of the mathematical sciences as practiced at the University of Arizona through projects that encompass both the "pure" and "applied" sides of mathematics.
- Promoting a multi-institutional intellectual culture that draws on the best practices in integrating research and education, both at Arizona and at partner schools, through the participation of faculty from those schools, as well as through the collaborative development of exportable project modules used during the program.
- Collaborations with guest faculty to develop programs of research experiences for students at their own home institutions through joint proposal writing and other cooperative ventures.

These goals will be achieved by drawing on the many unique components of our academic environment including our disciplinary and interdisciplinary research groups, our laboratories, and our education and outreach programs. Each summer a carefully designed, thematically based program will be offered by a faculty team assisted by advanced graduate students for whom the design and mentoring of the summer projects can constitute part of their VIGRE experience. Guest faculty from participating institutions will be an integral part of the program management team, contributing to lecture courses and project mentoring.Our local VIGRE postdocs will also be able to participate, where appropriate, in the summer program.

Logistically, a typical summer program will run for four weeks over the month of July during Arizona's spectacular summer monsoon season. This time-window will enable students from schools on both the semester and quarter systems to attend. The program will support up to twelve students who will be housed in University of Arizona dormitories, with local graduate student mentors providing essential social oversight and cohesion. Students from around the country, and especially the Southwest, will be invited to apply. Acceptance into the program will be based on academic criteria, and the student participants will be paid a stipend and living allowance. Guest faculty will be offered some living expenses and a summer stipend. We note that our summer program is not restricted to undergraduates in their junior year, and seniors, including those who have been accepted to other graduate programs, will be eligible to attend.

The scientific program will be complemented by ancillary activities reflective of the rich and diverse culture of the Southwestern United States, as well the wealth of scientific and educational activities in the region. For example, participants will be able to visit and participate in the Department's Summer Camp for American Indian students and other outreach activities including those sponsored by CEMELA; as well as make trips to Kitt Peak, Karchner Caverns, the Biosphere, etc.

Although there will be organizational variations between different summer programs depending on their mathematical themes, logistical needs (e.g., some will have a laboratory based component) and pedagogical philosophy, their basic structure of intense immersion and pro-active mentoring is the same. A fundamental goal of all the programs will be dissemination: material developed during the summer will be incorporated in future undergraduate courses, and be made available for use at the home institutions of the participating faculty, and in on-line format for use elsewhere. The design of the specific programs is based on many years of experience gained in developing related activities under our prior VIGRE grant, such as the modeling course, the laboratory based skills seminar, and our many outreach projects.

Below we describe the specifics of three programs that we intend to offer: in mathematical modeling, cryptography and number theory, and the mathematics of fluids. University of Arizona faculty and faculty from partner institutions have also agreed to organize several other summer programs on topics such as random matrix theory, mathematical biology, and computational group theory.

4.4.1 Summer Program 1: mathematical modeling

This program, based on our recently developed undergraduate mathematical modeling course described in Section 4.2.6, will be strongly team-based from the outset. Each team of students will choose a module from a list of proposed topics. These modules will be of two types: *review* or *case study*. A review module will consist of a self-paced study program: possible topics include planar dynamical systems, linear algebra, and programming in MATLAB. A case study module will be based on a research paper (typically published in Nature, Science, or the Proceedings of the National Academy of Sciences), which illustrates how mathematical models are used in the sciences. Both types of module will aim to produce descriptive text, figures, links to online information or applets, exercises, and MATLAB simulations (the latter will be compiled in order to be usable by students who do not own a MATLAB license) for use in future undergraduate modeling courses here and elsewhere.

The summer program will be organized as follows. In week one, participants, graduate mentors, and faculty coordinators get to know each other, and teams are formed and projects chosen within the first few days. In the following days, the participants start to work in their teams with assigned graduate student mentors and become familiar with the module they are going to develop. They start designing, with appropriate input from their mentors, the essential structure of their modules. Relevant material is researched online or at the library and collected for future use. The graduate mentors meet on a daily basis with the faculty coordinators and report on the progress of their team. This will be a time for each mentor to discuss problems or issues that may have arisen, get advice from their peers or from the faculty coordinators. At the end of the first week, the teams give an oral presentation describing the modules they are going to develop. Weeks two and three follow the same interactive format as above, with an emphasis on assembling the modules. The teams report on their work on the last day of each week, and receive feedback from the other teams, mentors and faculty coordinators. During the last week of the program the modules are completed, and the last day is used for group presentations, and an overall evaluation of the summer program.

4.4.2 Summer Program 2: cryptography and number theory

This program will illustrate the mutually reinforcing interactions between fundamental mathematics and real-world issues of implementation and practicality through modern cryptography. Students will be exposed first to basic number theory, then to its applications in private key ciphers, including the new Rijndael Advanced Encryption Standard. Then basic probability and information theory will be used to analyze the practicality and performance of the cryptographic protocols. Another iteration of this cycle will lead from more advanced number theory to public key ciphers such as RSA and El Gamal and on to quantum computing paradigms and Shor's factorization algorithm. The overarching goal is to illustrate the power of the mathematical ideas and the influence of concrete problems on the development of mathematics.

On an organizational level, this program will have three main components. Fundamental mathematics will be learned through lecture courses and highly interactive problem sessions in small groups with frequent student presentations and feedback. Implementation issues will be illustrated and explored via computer simulation using standard packages such as Mathematica or Maple in networked computer laboratories. Students will communicate their findings, both orally during the problem and simulation sessions, and in writing; projects will culminate in a written account of findings, including theory, implementation issues, and code, suitable for posting on our web site and for use in 400-level undergraduate capstone courses in cryptography. The cycle of theory, implementation, analysis, and write-up will be carried out over a period of two weeks, with two iterations over the course of the 4-week program.

4.4.3 Summer Program 3: the mathematics of fluids

This program will begin with an intensive course on elementary fluid mechanics and the principles of mathematical modeling (such as dimensional analysis and scaling). Different introductory text materials will be used to show the more theoretical approaches (*e.g.* Chorin and Marsden's "A mathematical Introduction to Fluid Mechanics") and more applications-oriented approaches (*e.g.* Acheson's "Elementary Fluid Dynamics") to fluid mechanics. Paralleling the course will be a series of sessions in the Applied Mathematics Laboratory introducing the students to data analysis techniques, as well as problem and review sessions run

by the graduate student mentors. Possible research projects, all of which have already been tested in other courses and which involve both mathematical and experimental components, will be introduced during the first week. They will include: (i) The pinch-off of fluid filaments—this project is ideally suited to illustrate the idea of singularities in solutions to equations and the mathematical consequences thereof. (ii) The coiling of viscous fluid filaments (such as honey)—this project is ideally suited to introduce students to the concepts of viscosity, and the modeling of the phenomenon using scaling arguments. (iii) The splashing of fluid drops—this visually entertaining project is rich in image and data analysis issues, and fluid mechanical concepts. (iv) The trickling of fluid down incline plates—this project offers many challenges in modeling and applications of dimensional analysis. A fundamental goal of all these projects is to introduce students to that most crucial of methodologies in applied mathematics, namely the interpretation and analysis of data, and its translation into self-consistent mathematical models.

In keeping with the general structure of all the summer programs, the groups will meet collectively on a regular basis to present their progress and discuss each other's progress. A timetable will be set for the students to produce a written report of their projects, and the each team will meet with a faculty mentor to discuss the evolving design of the report and the quality of the writing. The program will end with a mini-conference at which the projects will be presented and evaluated.

4.4.4 Summer program participants

A large number of regional institutions have already committed to participate in our summer program. These include including Harvey Mudd and Claremont McKenna Colleges, California State University at San Marcos, the University of Nevada at Las Vegas, the University of New Mexico and New Mexico State University, the University of Colorado at Boulder, the University of Texas at Arlington, the University of North Texas at Denton, the University of Texas Pan American and Texas A&M Corpus Christi, and the University of Nebraska. A few sample letters of commitment are included in our supplementary documentation.

These institutions represent a broad spectrum of educational approaches and student demographics. Tying them together in a mutually cooperative regional network will increase opportunities for students and faculty, achieve diversity goals, and enhance the pipeline of well-prepared mathematical scientists.

4.4.5 Relationship to other summer programs

There are many excellent summer programs around the country; ranging from traditional REU sites to professional development programs targeting specific groups, such as the NSF-supported EDGE collaboration between Bryn Mawr and Spelman to increase the success of women minority graduate students in graduate school. In particular, the IMMERSE program at the University of Nebraska, funded by NSF under the MCTP program, also aims to bridge the gap between undergraduate and graduate studies, and will provide a model for us to learn from as we develop our summer school. The University of Nebraska is also among the institutions that have expressed interest in taking advantage of our guest faculty program.

4.5 Achieving VIGRE goals: cross-cutting mechanisms for interaction, integration, and mentoring

The graduate, undergraduate, post-doctoral, and summer components of our VIGRE program are all woven together by the cross-cutting mechanisms of vertical and horizontal integration, and a system of individual and team-based mentoring processes. Each element of the training program described in the previous sections involves one or more components of multi-level interactions, integration of research and education, enhancement of communication skills, effective mentoring, and mechanisms to share our training paradigm with other institutions.

4.5.1 Mechanisms for interaction at different levels

Vertical integration between academic levels, and horizontal integration between different disciplines and research areas, has long been a hallmark of the mathematical sciences at Arizona. Virtually every aspect of our VIGRE proposal is designed to foster interaction in an intellectually coherent and sustainable fashion. Illustrative examples include: (i) The integration workshop (4.1.2) fosters interactions among faculty, senior

graduate students, and incoming graduate students in the context of core mathematical topics. (ii) Research Tutorial Group projects (4.1.3) foster topical and mathematically deep interactions between new graduate students and faculty, and can also involve the participation of post-docs and senior graduate students. (iii) The URA and UTA programs (4.2.7) foster interactions between faculty and undergraduates, and can again involve post-docs and graduate students. (iv) High school calculus class visits (4.2.3) foster interactions across the entire academic spectrum, from future undergraduates to senior faculty. (v) The vertically integrated operational structure of the summer program (4.4) fosters the mentoring skills of graduate students who play in intermediary role between the student participants and the faculty.

Our many research seminars are highly vertically integrated with participation of all ranks: from graduate students to senior faculty; and our collegially interactive culture allows for an unusual degree of horizontal integration. For example, last year a theme of the Analysis and its Applications Seminar, which is run by the Applied Mathematics Program, was "applied number theory" involving contributions from number theorists, computer scientists, and engineers. Indeed, cross-disciplinary interactions between faculty in different disciplines, often mediated by graduate students, have long been a hallmark of the Applied Mathematics Program at Arizona.

Interactions between Mathematics faculty and Mathematics Education Research faculty are often fostered by issues of content and pedagogy in the undergraduate classroom. The recent example concerning the concept of mathematical definition described in Section 4.2.5 has been valuable and may serve as a model for future interactions. The research of PhD candidates in Mathematics Education also naturally fosters interactions between groups.

4.5.2 Integration of research and education

The integration of research and education has also long been a tradition at the University of Arizona which we will continue and strengthen. Mechanisms for this integration include: (i) A wide range of innovative courses and programs, ranging from the Arizona Winter School for advanced graduate students and post-docs, to the technology-intensive business calculus course for Freshman developed jointly with the Business School. Students and post-docs have the opportunity to participate in all of these activities at appropriate points in their careers. (ii) The URA program (4.2.7) naturally integrates research and education at the undergraduate level. (iii) A comprehensive system of teacher training and mentoring, especially at the entry graduate level. For more advanced graduate students and post-docs, mentoring of teaching is done on a more individual basis, and Professors Deborah Hughes Hallet and William MaCallum have been particularly active in this context. (iv) Progression through through the TA ranks from Assistant I through Associate II, in both graduate programs, is predicated on satisfactory progress in *both* research and teaching – sending a clear message to our students that professional mathematicians must be able to create *and* communicate mathematics.

4.5.3 Communication skills

A comprehensive range of communications skills, both written and oral, is fostered at all stages and naturally integrated into the scholarly pursuits of the participants as described in the previous sections.

For students in both graduate programs these opportunities include: (i) written and oral presentations of term papers and research projects; (ii) VIGRE proposal and resume writing; (iii) research seminar presentations; (iv) presentations (posters and short talks) at national conferences and workshops; (v) teaching at different levels, including tutoring and outreach. Other communications skills requirements for the graduate students are described in the Curriculum Review Section.

For undergraduates the opportunities to improve communication skills include: (i) resume writing; (ii) presentations (posters and short talks) at workshops and meetings, including the Arizona Mathematics Undergraduate Conference (AMUC); (iii) URA presentations and UTA activities.

For post-doctoral fellows opportunities for enhancement of communications skills include: (i) research seminar presentations and organization; (ii) research proposal preparation; (iii) teaching at both undergraduate and graduate levels; (iv) outreach, mentoring, and advising opportunities including supervision of undergraduate research projects, co-advising of RTGs and Master's students, and participation in the summer program.

4.5.4 Mentoring and advising

Our system of individual and team-based mentoring described in the previous sections not only provides the appropriate guidance for VIGRE participants at different levels, but also provides mechanisms for developing natural professional relationships across these levels.

Graduate mentoring. Graduate mentoring starts at the recruitment phase and carries through until graduation. Over the years our experience has been that the most effective mentoring is provided by a combination of structured advising and the natural relationships that develop between students and faculty through intellectual engagement in classes and research projects and, ultimately, dissertation research. First and second-year graduate students in both programs are closely advised by their respective program heads through regular meetings. This degree of close attention from the program heads is maintained until the students have passed their qualifying exams. During the first (and second) year, the core course instructors often develop working relationships with the students, and these provide additional sources of advice and support. Furthermore, there is a long tradition of collegiality among all the graduate students with older students, as well as post-docs, playing natural mentoring and supporting roles for the new students. In the intermediate years, between completion of the core requirements and passing the oral comprehensive exams, the graduate program heads provide general oversight of the student while natural mentoring relationships are being developed with other faculty through the of the third-semester research projects, and other independent study activities—some of which are carried out with older students and post-docs. The VIGRE fellowship proposal writing and reporting system provides additional mentoring in professional development at this stage. During the dissertation phase we follow the time-honored tradition of assigning primary mentoring responsibility to the student's dissertation director. However, the program heads continue to monitor progress and become involved as needed to ensure reasonable times to degree.

Where appropriate, we also draw on the scholarly research devoted to mentoring practices. For example, ideas from the book *Entering Mentoring* [Han05] will be used to train graduate students mentors participating in some of the proposed summer programs.

Undergraduate mentoring. Each undergraduate mathematics major is assigned a faculty advisor and about half of our faculty serve in this capacity. We expect the advisors to meet with their advisees at least once each semester, and the Math Center makes a concerted effort to accomplish this goal through the organization of various faculty-student get-togethers. The Math Center is also actively involved in mentoring undergraduates while they are in the crucial "Introduction to Proofs" course, Math 323. At this time, students are helped to produce resumés and apply for internships. The Center also routinely checks on the progress of the junior and senior mathematics majors to ensure that they are progressing towards their career goals. In Fall 2004, the Mathematics Department began organizing luncheons for the invited colloquium speakers and undergraduate mathematics majors to foster connections between our students who wish to go on to graduate school and visiting faculty from Ph.D.-granting institutions.

Complementing the mentoring provided by faculty and Math Center personnel, is that provided by advanced graduate students and post-docs through their participation in various vertical integration activities and capstone experiences described earlier. These types of mentoring interactions provide the undergraduate students with attractive (and often less intimidating) role models while, at the same time, give the mentors valuable professional training.

Postdoctoral mentoring. For our postdoctoral fellows, the most effective mentoring (again) is provided by a combination of structured advising and the natural relationships that develop through intellectual engagement. Mentoring on career trajectories, appropriate course selection, outreach opportunities, etc., will be provided through regular meetings with the Department or Program Head, and by information sessions run by the director of the Teaching Post-doc program. Mentoring on research, publication, and other mathematical issues will be provided by the senior faculty in the research group(s) to which the postdoctoral fellows is attached.

4.5.5 Effect on unsupported graduate students and post-docs

Our VIGRE activities are integrated into the entire fabric of the Mathematics Department and the Applied Mathematics Program. Accordingly, they are open to all students in both graduate programs, independently

of whether they have VIGRE support. Our approach is highly inclusive and non-VIGRE eligible students and post-docs participate in, and benefit from, the entire spectrum of VIGRE supported activities. Where possible, other resources are used to support the professional development of non-VIGRE eligible individuals, e.g., travel funds from grants and institutional sources to attend conferences.

4.6 A rich infrastructure of research and educational opportunities

Our rich and diverse intellectual and organizational infrastructure contributes to many of the elements of our proposed VIGRE program. Below we list, with a brief summary, some of these key components.

4.6.1 Applied Mathematics and Biophysical Training Laboratories

The Applied Mathematics Laboratory is a one-of-a-kind experimental teaching laboratory that was developed to teach undergraduates and graduate students the principles of mathematical modeling through the generation and study of real experimental data that can give students an appreciation of the experimental issues connected with data collection and analysis, as well as the validity of the mathematical models themselves. The laboratory is equipped for experiments in condensed matter physics, material science, and fluid mechanics, and has facilities for high-speed photography and image analysis. The laboratory was started in 1995 with funding from the NSF Division of Mathematical Sciences. A second laboratory, the Biophysical Training Laboratory, also managed by the Applied Mathematics program, was started in 2000 with funding from an NSF IGERT grant to deliver the same experiences as the Applied Mathematics Laboratory in a biological setting.

4.6.2 ACMS

The Arizona Center for Mathematical Sciences is an interdisciplinary research group within the Department of Mathematics which enjoys close ties with the Departments of Physics and Optical Sciences. It is an internationally recognized center of research in nonlinear optics and provides outstanding research opportunities in areas such as nonlinear optics and telecommunications for graduate students and post-docs. ACMS receives substantial funding from the AFOSR and other agencies.

4.6.3 AIMES

The AIMES (Arizona Internships in Mathematics, Engineering, and the Sciences) program, managed by Mathematics faculty member Juan Restrepo, serves as an information clearing house for the many existing internship opportunities in government laboratories, industry, corporations, and academic institutions.

4.6.4 Arizona Summer Math Camp

The Mathematics Department has run a summer mathematics camp for mathematically gifted high school students for the past twenty years. Funded by a local foundation, this two week camp attracts students from around the country but concentrates on Arizona students. While it is primarily a part of the Department's community outreach efforts, it has also proved to be a very successful recruiting tool with quite a few summer camp alumni returning as successful mathematics majors.

4.6.5 Biology Mathematics and Physics Initiative

The Biology, Mathematics, and Physics Initiative (BMPI) is a multidisciplinary program, managed by the Program in Applied Mathematics, whose goal is to bring together graduate students, post-docs, and faculty from many departments to advance research and education in the biological and biomedical sciences through the interface with mathematics and physics. This program was started under an NSF IGERT grant and is now funded by the University of Arizona's BIO5 Institute for Collaborative Bioresearch.

4.6.6 CEMELA

The Center for the Mathematics Education of Latinos is a new, NSF-funded Center for Learning and Teaching involving a partnership between the University of Arizona, The University of Illinois at Chicago, The University of California at Santa Cruz, and the University of New Mexico, and various school districts. The objectives of CEMELA are to: (i) develop leaders in mathematics education; (ii) conduct research; and (iii) strengthen pre-service and in-service teachers' ability to promote Latinos' achievement in mathematics.

4.6.7 CRR

The Center for Recruitment and Retention of Mathematics Teachers at the University of Arizona was created in August, 2001 to address the shortage of qualified mathematics teachers at the middle and high school levels. Its goal is to attract college students into mathematics education and to retain those currently in the field. Outreach activities to the high schools that VIGRE participants engage in are coordinated by the Center.

4.6.8 Mathematics Education Program

This sub-program of the Mathematics Department is involved in scholarly activities, oversees the Department's teacher preparation programs, and carries out an array of outreach projects. It maintains a strong program of research and scholarly productivity, and has produced innovative, nationally acclaimed texts for college students. In research, the group focuses on teacher education and on issues of equity and diversity in the teaching and learning of mathematics, primarily within a socio-cultural framework. There is an increasing interest in research in mathematics education at the undergraduate level. The creation in 1992 of a mathematics education option in the Mathematics Department's Ph.D. program, and a teaching option in the MA program, complement these research activities.

4.6.9 Southwestern Center for Arithmetical Algebraic Geometry

The Southwestern Center was established in 1997 with funding from an NSF Group Infrastructure Grant; further support was obtained from the NSF in 2002. The most important activity of the center is the annual Arizona Winter School, an intensive, five-day meeting intended mainly for advanced graduate students and post-docs which features an integrated set of courses on an advanced topic by leading experts. In contrast to a typical conference where individual researchers present their work in relative isolation, the AWS features a small number of extended courses on a set of closely related topics. Students work on projects with the lecturers during the Winter School and present their results during the meeting. The Winter School puts great emphasis on ensuring significant interaction among all participants, especially between students and the senior lecturers.

4.6.10 Teaching Post-doc Program

The Teaching Post-Doc program provides a three-year professional training position which is similar to post-doctoral positions in research, except that here the emphasis is on teaching and scholarly activities pertaining to teaching. These post-doctoral positions are part of a professional training program designed for faculty of mathematics planning to enter a career in higher education. The program has been highly successful, especially in integrating professional development activities with the more standard post-doc responsibilities.

5 Recruitment and Retention

Many of our current and proposed recruitment and retention mechanisms have already been described in Sections 4.1 through 4.4. Here we give more details, and discuss specific efforts directed towards women and minorities. The VIGRE project is, of course, primarily directed at U.S. citizens, nationals, and permanent residents, but the inclusive nature of all our activities does not discriminate against foreign students. They are also able to benefit from our recruitment and retentions efforts and, indeed, sometimes become U.S. citizens or permanent residents themselves.

5.1 Recruitment

In Fall 2003 the Mathematics graduate program initiated an aggressive new campaign to increase the number of high-quality applicants. We completely overhauled the graduate section of the Mathematics Department's web site, which now includes an on-line inquiry form, and made concerted efforts to identify and contact prospective applicants through such means as NSF REU sites, undergraduate research conferences, and poster sessions at the national meeting. We now engage prospects with much more personal contact than had previously been the norm: prospective students receive a letter from the head of the graduate program detailing the unique features of the program, another from a faculty member in the broad research area the prospective applicant has identified as their area of interest, and a letter from a current graduate student about life in the Department and in Tucson. So far these initiatives have been very successful and have helped us recruit excellent classes over the past two years. The Applied Mathematics Program has maintained an active and successful recruiting program for many years using a combination of strategies including on-line tools, mailings of various kinds, and phone calls. Program alumni in academic positions often direct undergraduates in their home institution to apply here.

The graduate programs in Mathematics and Applied Mathematics have for almost two decades operated a joint recruiting workshop in early spring. Typically 20–25 applicants to the two programs are invited to campus for a three-day event that includes lectures on current research in the Department and the Program, meetings with faculty and current students, and various social activities. Students get an in-depth view of the programs and conversely, we learn much more about the students. The very fact that this workshop is a joint effort immediately sets the tone for collegiality between the two graduate programs.

Looking forward, our new summer school initiative (Section 4.4) is designed to increase the flow of students into top-tier graduate programs, including our own. By giving students a taste of authentic research in mathematics, and of the broad spectrum of opportunities available here, we will motivate them to pursue advanced study and careers in the mathematical sciences. The involvement of faculty from regional institutions, many of which serve under-represented groups, will only help in this process of building the pipeline of students from diverse backgrounds into the profession.

At the undergraduate level, the Math Center has orchestrated an aggressive campaign to increase both the number and quality of our undergraduate math majors. This effort extends from the high school level (see the discussion of the Calculus classroom visits and the high school math workshops in 4.2.3), through contacts with students in lower division course work (see 4.2.4), to the "transition to the major" course Math 323. The quality and diversity of our professional development activities (URA, UTA, Internships) are also effective tools for recruiting into the major. Over the last 3 years, the Department has had on average over 375 mathematics majors of which about 60 are minority students.

At the post-doctoral level, we have found it difficult, as have many other VIGRE sites, to find and compete for highly qualified US citizen post-doctoral fellows. Nonetheless, we have had some successes and our recruitment strategies include developing more connections with the Teaching Post-doc Program (4.6.10) in the Mathematics Department, and further promoting the opportunities for interdisciplinary activities in the Program. The large number of professional development activities that have been developed as part of our overall program will add an attractive extra dimension to our efforts to recruit VIGRE post-doctoral fellows.

5.2 Retention

A recent survey of the literature on retention of graduate students in PhD programs in mathematics drew the following conclusion:

> "... The key to retaining diverse students in graduate mathematics is to develop ways to help students participate fully and effectively in authentic mathematics practice." [Her04, p. 202]

Here by "authentic mathematics practice," the author means things that mathematicians actually do (struggle with formulating and proving conjectures or constructing models and analyzing them, communicating through extended writings or lectures, etc.) as opposed to typical student activities such as listening to a long series of lectures or solving problems that have already been broken down into small steps. Virtually every aspect of our VIGRE project directly addresses this conclusion: activities like the RTGs, term papers, the Integration Workshop and first-year Skills seminar at the graduate level, and URA, UTA, and Internship programs at the undergraduate level, are designed to involve students in professionally realistic research and communication activities as early as possible. Thus by its very design, our VIGRE project speaks to the retention issue.

Retention is further supported by our mentoring structures and multi-component assessment mechanisms that ensure that performance is comprehensively evaluated. For students in the Mathematics program these efforts begin with the Integration Workshop and continue with regular meetings between the students and the Associate Head for the Mathematics Graduate Program—who also meets regularly with the core instructors and RTG sponsors to identify and head off potential problems before they become serious. Similar retention efforts are made by the Applied Mathematics Program with active oversight by the Program Head of the students' progress.

At the undergraduate level, the Associate Head checks enrollment of the upper division classes that the mathematics majors take and monitors the grades of the students. For those students who are performing well, he tries to find them internship opportunities. For those students who are having difficulty, he meets with them to identify what the problem is and to determine how best to proceed in order to complete their degree. The Math Center will begin this year to invite our past graduates to give talk to undergraduates about career opportunities that are available for mathematics majors.

5.3 Women and minorities

At the graduate level, the Associate Head for the Mathematics Graduate Program has brought new efforts in recruitment and retention of excellent minority and female students. In particular, cooperation with the University of Arizona Graduate College provided significant recruiting funds for women and minorities for the last two recruitment seasons. Similar efforts have been made by the Head of the Applied Mathematics Program. We have also started sending faculty members, and current students, to events such as the annual meeting of SACNAS (the Society for the Advancement of Chicanos and Native Americans in Science), the Blackwell-Tapia conference at IPAM, and the George Washington University summer program for women in mathematics to advertise both our graduate programs and recruit students. As an illustration of our efforts we note that in 2004, 5 of the 13 incoming graduate students in Mathematics, and 6 out of 9 incoming students in Applied Mathematics, were women.

Retention efforts include careful selection of core instructors and participants in the Integration Workshop to ensure suitable support for students early in their programs, careful monitoring and mentoring of the new students by the program heads, and lectures by high-profile minority and female scholars, and by experts in recruitment and retention of under-represented groups in mathematics. Where necessary, senior graduate students are assigned as "personal tutors" to help individual students through the rigors of the core courses.

At the undergraduate level, for more than 15 years Associate Head Velez has run his calculus advising program in which he attempts to meet with all minority students enrolled in the three-semester calculus sequence to provide support and advice. He encourages many of them to either major in mathematics, double major in mathematics, or at least minor in mathematics. Because of these activities the number of minorities majoring in mathematics has increased over the years. Additional programs to help bring minority school children to the University of Arizona and interest them in mathematics include the newly established Center for the Mathematics Education of Latinos (CEMELA), and long-running summer program for Native Americans from the White River Apache reservation.

6 Performance Assessment

Our assessment plan was developed in collaboration with Dr. Victoria Cunningham of the University of Arizona's Office of Instructional Assessment. In the first two parts of this section, we outline the goals of our project and our standards for success. In the remaining section, we detail the evaluation plan for our project. If the proposal is funded, we will work with Dr. Cunningham to implement this plan. (A statement of cooperation is attached.)

6.1 Goals of the Project

The goals of our VIGRE project are to:

1. Train mathematical scientists at all levels who are well-prepared for research, communication, and mentoring, and who are therefore well-prepared to continue their careers in the mathematical sciences. At the graduate level, our students will have:

   - Broad and deep training which combines solid foundations with significant contributions to the mathematical sciences.
   - Authentic research and communication experiences as early in their programs as possible.
   - Unparalleled opportunities for interdisciplinary research.
   - Quality teaching skills fostered by carefully mentored teaching experiences at a variety of levels.

   At the undergraduate level, our students will have:

   - Solid training in core mathematics or in one of the more applications-oriented tracks of the major.
   - A capstone experience involving significant research and writing.
   - Individual research (URA), teaching (UTA), and internship experiences.
   - Ample interactions with more advanced mathematical scientists, ranging from graduate students through senior faculty.

   Our post-doctoral participants will have:

   - A mature research program developed under the mentorship of senior faculty members.
   - A variety of teaching experiences from the basic undergraduate level through the advanced graduate level.
   - Mentoring and outreach experiences with high school students, undergraduates, and graduate students.

2. Increase the number of participants who go on to high quality mathematical sciences careers, defined as follows:

   - graduate students will obtain post-docs at group I or II institutions, research institutes, or national labs, or tenure-track positions, or technical jobs in industry.
   - undergraduates will continue on to graduate school in mathematics or other scientific or technical discipline or to a professional school.
   - post-doctoral participants will obtain permanent positions in academia or technical jobs in industry.

3. Improve the recruitment and retention of and climate for women and under-represented minorities.

4. Make a permanent change in the workforce training infrastructure by integrating VIGRE activities into the intellectual and administrative life of our programs and by implementing our post-VIGRE plan. This fundamental goal will be achieved by:

   - Ten new long-term post-VIGRE graduate fellowships from the University of Arizona, to be phased in starting in the third year of the grant. (Letter of commitment attached.)

- Transferring funding for our undergraduate research program from VIGRE to endowed funding and federal undergraduate research funding over the course of five years.
- Transferring funding for our undergraduate teaching assistants program from VIGRE to endowed funding over the course of five years.

6.2 Standards for Success

Progress towards our goals will be measured against explicit, quantitative standards. These goals were arrived at after an analysis of our performance over the last 10 years (see Section 2) and in many cases represent significant improvements over national norms.

1. Effectiveness at training well-prepared students and post-docs is hard to measure directly, but there are a number of proxy standards which can be used:
   - Participants will have a high level of participation in integration activities and a high level of satisfaction with the outcomes. This is essentially a measure of the experience participants while they are in the program.
     - all VIGRE-funded graduate students and post-docs will participate in at least one integration activity per funded semester or summer.
     - at least 50% of graduate students and post-docs will have had a meaningful professional development experience (e.g., speaking at a conference, doing an internship, writing a joint paper with a faculty member) before leaving the program.
     - at least 50% of undergraduate mathematics majors will participate in a capstone, URA, UTA, or internship experience by the time they graduate.
     - at least 90% of participants will find the integration activities, capstones, URAs, etc. they participated in to be valuable experiences.
   - A long-term measure of outcomes will be provided by career trajectories of participants. This is also part of goal number 2 (the "pipeline" goal) and is discussed more below.
2. A high percentage of participants will continue on to more mathematical training or careers in the mathematical sciences:
   - at least 50% of entering PhD students will obtain a graduate degree in 6 years or less.
   - at least 85% of PhD graduates will leave the program with employment in the mathematics sciences.
   - at least 90% of PhD graduates will publish a paper related to their dissertation in a reputable journal (to be defined in the evaluation phase) within 3 years of graduation.
   - at least 25% of undergraduate mathematics majors will continue on to advanced degrees in mathematics, science, engineering, or the professions.
   - at least 50% of Arizona Summer School participants will enter graduate school in the mathematical sciences.
   - at least 80% of post-doctoral fellows will obtain a position in a US university or a mathematically-intensive job in industry or one of the national laboratories.
3. On female and minority participation:
   - on average, at least 33% of PhDs will be awarded to females; this would be near the top of the national range.
   - on average, at least 10% of our PhDs awarded to US citizens will go to minorities; this is about twice the average rate nationally.
   - on average at least 40% of participants in the new summer program will be female and at least 25% will come from under-represented minority groups.

- every effort will be made to recruit female and minority post-docs. This is not a quantitative goal—the global numbers are simply too small to set quantitative goals.
- on average, at least 35% of undergraduate degree recipients will be females and at least 20% will be underrepresented minorities. A variant of this goal, currently equivalent, but perhaps better for long term comparisons, is that the percentage of minorities among our undergraduate degree recipients will be greater than the percentage of minorities in the overall University of Arizona undergraduate population.
- on average there will be at least two visits per semester by experts on diversity, recruitment, and retention issues, or by high-profile minority or female mathematicians. These visitors will lecture in one of our three weekly colloquia. A further, non-quantitative goal is that these visits will have an impact on our programs and policies.

4. The standards for our post-VIGRE plan are evident: we will implement the plan described above and in Section 9.

We emphasize that these are not predictions of the future—they are standards against which our efforts will be measured.

6.3 Evaluation Plan

Our evaluation plan is summarized in two complementary tables. The plan's phases and guiding strategies in Table I include (i) assessing our current effectiveness at training mathematical scientists, ensuring that they continue on to mathematical careers, and recruiting and retaining females and minorities; (ii) monitoring the implementation of our VIGRE program and providing feedback for possible program alterations; (iii) evaluating the effectiveness of the program in meeting the goals outlined above. The evaluation plan in Table I is divided into the following three interrelated phases:

- **Needs Assessment Phase.** First, we will examine the academic and career histories of students and post-docs who participated in our first round of VIGRE-funded program activities to provide baseline data. We will analyze these data for influences of VIGRE activities on student academic progress and outcomes, especially within the realm of career choices and opportunities. Second, we will use program information and course evaluations to establish baseline conditions for the curriculum component of the program. Finally, we will query program staff, faculty, and students about student utilization of existing VIGRE-related services for guiding program activities to meet the needs of the students.
- **Implementation and Monitoring Phase.** This formative phase of our evaluation plan includes strategies for monitoring program implementation (process evaluation) and comparing implementation with plan (program fidelity), in order to understand what program components are actually delivered and when, and for feedback to VIGRE staff to alter the program if necessary. This phase will extend up to the three-year review of the grant. It will include ongoing, strategically placed assessment activities aimed primarily at identifying strengths and weaknesses of all VIGRE components.
- **Outcomes Phase.** The outcomes or summative evaluation phase will include intermediate as well as long-term outcomes. It will focus on the efficacy of the program and on whether the program's objectives were met in terms of impact on all VIGRE participants.

The summary of program objectives and evaluation strategies in Table II maps our evaluation plan to the goals of the project. The first four rows correspond to goals 1 and 2 above (increasing preparedness and numbers), broken out by level in the program (undergrad, beginning grad, advanced grad, and post-doc) and the last two rows correspond to our recruitment/retention and post-VIGRE goals.

Table I. Evaluation Plan Phases and Strategies			
INFORMATION NEEDED	**SOURCE OF INFORMATION**	**MEASURES**	**INFORMATION UTILIZATION**
Needs Assessment Phase			
Participation levels and satisfaction	Math, Applied Math, Student Information System, Teacher Course Evaluations, participants	Enrollment rates, Teacher Course Evaluation results, satisfaction levels	Establish baseline
Previous participants' career paths	Mathematics, Applied Mathematics, participants	Graduation rates and time to degree, exit interview results, post-program tracking	Establish baseline
Female and minority recruitment and retention	Mathematics, Applied Mathematics, participants	Satisfaction levels, participation rates in program activities	Establish baseline
Implementation/Monitoring Phase			
Effectiveness of recruitment efforts at all levels	Academic advisors, undergraduate and graduate students, post-doc fellows, program staff	Number of students inquiring, applying, and enrolling, perception of program information	Identify strengths and weaknesses of recruitment efforts
Faculty progress in meeting needs of participants	Faculty, students, post-doc fellows, program staff	Survey and interview results	Identify program strengths and weakness, track mentoring effectiveness
Participants' progress	Faculty, undergraduates, graduates, post-docs	Observation, results of interviews, student evaluations, grades, rates of retention and graduation, extra-curricular activities	Assess impact of VIGRE program on students' level and engagement, develop student profiles
Effectiveness of training for mathematical careers	Faculty, students, post-doc fellows	Results of course and workshop evaluations, placement rates	Assess quality and utility of training curricula
Outcomes Phase			
Participants' academic progress	Faculty, students, post-doc fellows, Student Information System	Interviews, observation, grades, placement	Assess impact of VIGRE-related activities on academic achievement
Teaching training effectiveness	Faculty, students, post-docs, Teacher Course evaluations	Teacher Course Evaluation results, placement	Assess improvement in training students to become effective teachers
Research training effectiveness	Faculty, graduates, post-docs	Placement, publications, funding, conference presentations	Assess improvement in training students to become effective career mathematicians

Table II. Evaluation Plan	
OBJECTIVE	**EVALUATION METHOD**
Improve preparation and numbers of Mathematicians: Undergraduate recruitment and retention	• Track outreach activities in local high school • Track participation in targeted "service" courses and other activities for recruiting mathematics majors • Monitor transition into and experience within upper division work by tracking participation in capstone, URA, UTA, or internship experiences • Track continuation in mathematical careers
Improve preparation and numbers of Mathematicians: Foundations for research and teaching	• Document utility of Integration Workshop and First-year Professional Development Seminar • Monitor students' progress in core courses, Research Tutorial Groups, term papers, and qualifying exams • Monitor mentoring of first teaching experiences • Monitor effectiveness of the Arizona Summer School
Improve preparation and numbers of Mathematicians: Graduate student training and professional development	• Monitor schedule of passing comprehensive exams and timeliness of students' "road-map" • Monitor participation in seminars, research conferences, publications • Track participation of graduate students in integration activities, internships • Get feedback from students about teaching experiences, mentoring • Track placement and post-program career
Improve preparation and numbers of Mathematicians: Professional development of post-doctoral fellows	• Monitor post-docs research development, publications, conference contributions • Track participation of post-docs in integration activities • Track internship and outreach activities • Track placement and post-program career
Improve Recruitment and Retention of Minorities and Women	• Monitor program recruitment activities (colloquia delivered by high-profile lecturers on diversity & retention, program representation at conferences targeting women and minorities) • Survey minority and women students on their motivation and satisfaction • Track number of PhDs awarded to minority and women students • Track number of undergraduate students who are minority and/or women
Establish permanent post-VIGRE training infrastructure	• Monitor progress of University in meeting commitments • Track private fund-raising efforts • Track REU supplement awards

7 Organization and Management Plan

The management and organization of the entire program will be achieved through a combination of equitably shared individual assignments and team work, and will utilize the administrative resources of the Mathematics Department and the Program in Applied Mathematics.

7.1 Management structure

General management and oversight of the VIGRE program will be carried out by the VIGRE committee which consists of the principal and co-principal investigators:

- Douglas Ulmer, Associate Head for the Graduate Program, Department of Mathematics, PI.
- Michael Tabor, Head, Program in Applied Mathematics, co-PI.
- Nicholas Ercolani, Head, Department of Mathematics, co-PI.
- William McCallum, Professor, Department of Mathematics, co-PI.
- William Velez, Associate Head for Undergraduate Affairs, Department of Mathematics, co-PI.

who between them have many years of experience in academic administration. Aspects specific to the three degree-granting programs (math graduate, applied math graduate, and undergraduate) will be administered by the corresponding Head or Associate Head, in collaboration with the program's governing committee. The post-doctoral program will be administered by the Head of the Mathematics Department and the Head of Applied Mathematics, in collaboration with the Department's personnel committee and a specially constituted committee to evaluate Applied Mathematics post-doc candidates. Many of the committees are partly elected and partly appointed, and care is taken to ensure that they have representation from a variety of research specialties and academic ranks. This structure will ensure broad faculty knowledge of and participation in the VIGRE program.

A number of other faculty members have agreed to oversee (in consultation with the management committee) other aspects of the program which are relatively complex or important or both.

- The VIGRE committee will have direct oversight of the Arizona Summer School initiative. Responsibilities include selection of the Arizona faculty member organizing the school in a given year, assistance with recruiting guest faculty, evaluation of the program, and dissemination of program results.
- Minority and female recruitment and retention initiatives will be managed by Joceline Lega and Joseph Watkins, with support from VIGRE committee members William McCallum and William Velez. Lega, a tenured female faculty member with broad research and teaching interests, has mentored a large number of female graduate and undergraduate students. McCallum chairs a University of Arizona College of Science committee charged with encouraging implementation of the Millennium Report on the status of women and minority faculty, as well as being a nationally recognized leader in undergraduate education and curriculum development. Velez is a nationally prominent figure in minority recruitment and retention, a former president of SACNAS, and the recipient of many awards for mentoring, including the President's Award for Excellence in Science, Mathematics and Engineering Mentoring in 1997. Watkins has extensive experience with Hispanic and Native American communities in Arizona, for example through the Native American Summer Institute. Examples of initiatives this team will be responsible for include ensuring strong minority and female participation in the summer program; sending faculty to meetings like SACNAS, the GWU program for women in mathematics, and the Blackwell-Tapia event at IPAM to recruit female and minority candidates to our graduate programs; and identifying and contacting potential speakers with expertise in recruitment and retention issues.
- The Undergraduate Research Assistant (URA) program will be managed by Robert Indik, who has managed this program since its beginnings in 1996. He is responsible for finding faculty sponsors, making students aware of the opportunities (e.g., through the undergraduate research seminar), shepherding students through the application and reporting processes, and selecting students for funding.

- The Undergraduate Teaching Assistant (UTA) program will be managed by David Lomen. Lomen is a Distinguished University Professor known for many curricular innovations, especially around the use of technology in the classroom. He will primarily be responsible for recruiting and mentoring a teaching post-doc to run the day-to-day mechanics of this program.

- Liaison with industry and the national labs for internships will be managed by Ildar Gabitov, Juan Restrepo, and Mac Hyman, with assistance from VIGRE management committee member William Velez. Gabitov has extensive connections with Los Alamos National Lab and was formerly employed there. Restrepo has extensive connections to Los Alamos, Argonne, and Sandia national labs and he has run for several years a program (AIMES) for connecting students with internship opportunities at companies. Hyman is a senior figure at LANL, the past president of SIAM, and an adjunct faculty member in the Mathematics Department. This team will generate leads for internships and ensure that students are aware of the opportunities for and benefits of internships.

- Liaison with the Mathematics Education Research community will be managed by Marta Civil. Civil is a widely known expert in Mathematics Education, specifically language and culture issues in elementary and middle school mathematics education. She is the PI of a new $10 million NSF grant funding a Center for Learning and Teaching at the University of Arizona.

- Outreach to local high schools will be overseen by Fred Stevenson and the Center for the Recruitment and Retention of Teachers. Stevenson has been involved in outreach in Arizona for over three decades and has run summer camps for Native Americans and gifted high school students for over 15 years. He will primarily be responsible for recruiting and mentoring a graduate student to run the high school workshops program and a teaching post-doc to run the high school visits program (see 4.2.3).

- Post-doctoral professional development will be overseen by Elias Toubassi. Toubassi has for several years managed the Department's extremely successful Teaching Post-Doc program. He will ensure that VIGRE post-docs are made aware of and take advantage of the spectrum of professional development activities that have been put in place as part of the VIGRE and teaching post-doc programs.

7.2 Evidence of Faculty Commitment

The structure outlined above ensures broad participation in the management of our VIGRE project: large numbers of faculty participate in the governing committees of the various units; special subcommittees have been constituted to oversee important subprograms; and the principal investigators have a track record of managing programs involving the efforts of large groups of mathematical scientists and educators—for example the Program in Applied Mathematics, the Southwest Regional Institute in the Mathematical Sciences (SWRIMS), Southwestern Center for Arithmetical Algebraic Geometry (SWC), and an IGERT program in biomathematics.

Broad participation in the intellectual life of our VIGRE program is also ensured: RTG, URA, and UTA projects always involve faculty, as do the Integration Workshop and First-Year Professional Development Seminar and the various outreach activities. For direct evidence on this issue, we refer to our results from prior support section where it is pointed out that over 78% of our tenured or tenure-track faculty have been involved with at least one significant aspect of our current VIGRE project. This substantiates our claim that our VIGRE activities are naturally integrated into the intellectual and professional life in the mathematical sciences at Arizona.

7.3 Relation to the RTG proposal

The University of Arizona is also submitting an RTG proposal to support the Southwestern Center and the Arizona Winter School. Although the two projects are quite distinct in their aims and methods, there are mutually beneficial interactions between them which we have already noted in our proposed project. If both projects are funded, the respective management teams will ensure, as they have for the last 5 years, that synergies are exploited and funds are used strategically to advance the aims of both projects.

8 Dissemination

We view "dissemination" as an opportunity to promulgate our best practice—which at Arizona is a whole culture of vertical integration between all levels of mathematician, and harmonious horizontal integration across all fields of mathematics. Our new summer program epitomizes this culture and will be a major dissemination vehicle for our VIGRE Program.

We see dissemination taking two main forms: informational - corresponding to specific information, such as technical reports and data on particular vertical integration projects; and material - corresponding to mechanisms for, and approaches to, research and education in the mathematical sciences. The internet plays, of course, a central role in disseminating both types of information, and we plan to use a dedicated web-site to present the successes and challenges of our VIGRE project.

Informational items to be posted include:

- An executive summary of the project, discussing goals, planned activities, management structure, and the post-VIGRE plan.
- Frequently updated pages advertising upcoming VIGRE opportunities and events, including the new summer program.
- Annual summaries of successes and problems.
- Pertinent information about our evaluation and assessment strategies
- The text of this proposal and all reports to NSF.
- Links to stories in the Mathematics and Applied Mathematics newsletters (which appear approximately twice a year and are on line) about VIGRE .
- Links to other VIGRE sites and sources of VIGRE information.

Material items to be posted include:

- Descriptions of each year's summer program.
- Summer project modules and an analysis of their effectiveness and implementation.
- Research reports on the modules written by the student participants.
- Software and other tools developed for the projects.
- Information about the Applied Mathematics Laboratory, and how to integrate experimental components into mathematical course work.

The AMUC conference (see Section 4.2.7) will be used to disseminate results of undergraduate research projects, as well as best practices in recruiting and retaining undergraduate mathematics majors.

Mathematicians at Arizona have been very active in DMS activities related to VIGRE, above and beyond participating at several meetings: McCallum and Tabor contributed two articles to the proceedings of the May, 2002 VIGRE meeting in Reston; Ercolani and McCallum contributed sections to the report from an April, 2003 meeting of Western VIGRE sites; and Ulmer made a presentation to the AMS Committee on Education in October, 2004 on the prospects for long-term change under VIGRE. Faculty involved in our VIGRE project will continue to contribute to the national dialog around VIGRE and more generally around workforce training issues.

Arizona was also active in helping to organize and contribute a regional VIGRE meeting which took place at the University of Washington in April, 2003, the report of which was mentioned above. We would be interested in organizing and participating in similar events which serve to identify and spread best practices in mathematical sciences workforce training, including regional meetings to discuss the experiences learned from the summer program.

9 Post-EMSW21 Plan

Our VIGRE program is carefully designed to integrate *naturally* into the intellectual and professional pursuits of all participants: undergraduates, graduate students, post-docs, and senior faculty members all participate in vertically integrated activities crafted to enhance their capabilities as mathematical scientists, educators, and mentors. Funding plays a crucial role role in this process: it relieves graduate students of heavy teaching burdens thereby allowing them to explore a greater range of career broadening opportunities; it gives undergraduates engaged in research and teaching a greater sense of professionalism, as well as providing valuable summer support; and post-doctoral funding provides an opportunity to give the recipients broader opportunities than would otherwise be available through either research-grant funded positions, or instructional appointments.

Accordingly, we have arranged for funding for the cornerstones of our VIGRE project to continue beyond the term of this grant. The most important aspect is fellowship funding for graduate students. The University of Arizona has agreed to put in place, by the 2010-2011 academic year, a total of 10 graduate fellowships, with a phase-in plan starting in the third year of the grant. (See the attached letter of commitment from the Vice President for Research.) The post-grant fellowships will be used in exactly the same manner as our current VIGRE fellowships: to provide time for students to deepen their education and hone their professional skills through vertically integrated research, teaching, and outreach activities including our summer program.

Another cornerstone of our VIGRE project is the undergraduate research assistantship (URA) program. We will fund the continuation of this activity through an endowed memorial fund (to be named after a faculty soon to retire who will not be named here), and by REU supplements to research grants of senior faculty members. Specifically, each main research group will be charged with generating URA project proposals and putting forward faculty to supervise them. This will result in a continuation of the highly successful URA program via a creative use of existing NSF programs. We will start implementing this mode of funding immediately, and accordingly have planned a phase-out of the URA portion of our budget over five years, to be offset by a phase-in of REU supplement funds.

A third cornerstone of our VIGRE project is the undergraduate teaching assistantship (UTA) program. To continue this activity, we have established a memorial scholarship fund, the Lusk fund (in honor of the parents of one of our undergraduate alumni) to be used to fund undergraduate teaching assistantships. The current balance in this fund generates enough income to fund a small number of UTA projects each year and we have plans to augment this balance through fund-raising initiatives over the next five years. Accordingly, we have planned a phase-out of the UTA portion of our budget over five years, to be offset by a phase-in of scholarship funds, or, if necessary, departmental funds.

The long history of cooperation between the two graduate programs, and their long-term commitment to VIGRE goals, will ensure that the necessary management structure of the post-VIGRE program will be effective. The management and distribution of the post-VIGRE graduate fellowships will be overseen by a joint Mathematics/Applied Mathematics committee whose chairmanship will alternate each year between the heads of the two graduate programs.

In conclusion, the intellectual opportunities and mentoring and support structures of our program are in place and well-tested. Our post-docs, graduate students, and undergraduates will continue to participate in vertically integrated research, teaching, and outreach activities which improve their training and skills as mathematical scientists, educators, and mentors. The financial plan outlined above will provide the resources to allow these activities to continue well beyond the life-time of the grant. Thus the net result of funding this proposal will be to make a valuable and permanent addition to the mathematical sciences workforce training infrastructure in the US.