ADVICE ON LEARNING MATHEMATICS

SEAN HOWE (SEANPKH@GMAIL.COM)

The purpose of this document is to give some concise advice on learning mathematics, targeted in particular at beginning undergraduates in mathematics. It consists largely of things that seem obvious to me now, but were not at all clear at the beginning of my career as a math major. Many of the ideas and advice contained in this document have their provenance in other documents of this nature or in advice I have received directly, but I have made no attempt to provide specific references – however, I will note that I have been particularly influenced by the advice on Terry Tao's blog, which I recommend to anyone pursuing the serious study of mathematics.

0.1. **Mathematics will eventually be difficult for you.** The only question is when. Different people have different aptitudes for mathematics, and if you've chosen to major in mathematics, there's a good chance that you are among those who up until this point have been "good at math." But what does it mean to be "good at math"? In elementary school it means being good at calculations, and in high school usually it boils down to having an intuitive understanding of the abstraction from specific equations to algebraic equations involving variables. Being good at calculus requires a new set of skills, and as you advance further in mathematics, what it means to "be good at math" will change again and again – for example, if you continue into theoretical math you will soon begin proof-based courses, and writing proofs is a skill almost completely independent from anything you have done before and there is no reason someone good at doing calculations or algebra or even calculus will be good at writing proofs.

The point is, nobody is really "good at math" because being "good at math" means too many different things depending on where you are in your education. Even if you thought all of your high-school math courses were simple, you may find yourself struggling in your first year math classes at university. Some of you will still cruise by, making it to your second or even third year courses before you encounter something that is truly difficult for you to understand. A few of you may go even further before you struggle, however, anyone who continues long enough in mathematics eventually reaches a point where the complexity of the concepts and tools involved renders them difficult to learn. It is important to understand that this is natural, and that ultimately your own success in mathematics depends much more on how you handle yourself when you reach this point than on how long it takes you to reach this point. Mathematics, in the end, is hard work, but if you're willing to do that hard work, then you will eventually be able to understand even the concepts that seem the most difficult to you when they are first presented. This brings me to my next point,

0.2. **Find something you love about mathematics and focus on that.** Nobody loves everything about mathematics, not even your professors. However, I

guarantee you that almost all of them (and all of the truly happy ones) love something about mathematics - whether it is the beauty of the structures that arise, the thrill of being on the cutting edge of research, the serendipitous moment when a clever problem opens your eyes to a whole new world of understanding, or the joy of seeing your work help someone, there is passion at the heart of it. When you reach the moment where things become difficult, if you do not love what you're doing then you won't want to do it anymore. For example, if you start taking proofs based courses (such as 215, 323, 413, 415, 425 at the U of A) and they are difficult for you, then only continue doing pure mathematics if you love something about pure mathematics – otherwise you'll have a hard time putting in the work necessary to overcome this difficulty. If you reach this point and you don't love pure math, start looking at other areas such as applied mathematics – there are many beautiful and interesting realms of mathematics where proof skills are less important, and you may find something that you love there.

Remember that not everybody does have or can find something they love about mathematics, even sometimes people who are "good at math" for a long time. If all the basics skills of mathematics come naturally to you, then it is particularly important that you make sure you find something you love early on, because when it does become difficult for you (see above), if you don't love it, then you may not want to do it any more, and this moment could come very late in the game leaving you in an awkward position. No matter what you end up doing with the rest of your life your education in mathematics will be useful to you, however, you'll be happier sooner if you're aware of what it is that you enjoy in what you do, whether it's mathematics or something else.

How do you find something that you love about mathematics? The best way is to spread as wide of a net as possible. Open yourself up to mathematics – take diverse courses, join your school's math club, talk to other students about mathematics. One of the best ways is to talk to your professors about mathematics – if a professor mentions something in a course that you think might be interesting to know more about, then go in to their office hours and ask about it! Explore mathematics by following your interests – if you're interested in computers, try to find out where math is used in computing (cryptography, 3d graphics, algorithm design, complexity theory, etc.), if you like sports, try to find out where math is used in sports (statistics, rankings, etc.). There are also several books aimed at presenting interesting mathematics – for example, "The Drunkard's Walk" by Leonard Mlodinow, which is accessible to people at any level of mathematical knowledge, or, if you are looking for a more advanced introduction to a variety of interesting topics in pure mathematics, try "Mathematical Omnibus" by Fuchs and Tabachnikov.

0.3. **Learn how to learn from different sources.** In any course you will have many sources of information available – lectures, textbooks, teaching assistants, classmates, problem sets, office hours, Wikipedia, etc. Some of these will be easier to learn from for you than others, and it is natural to lean heavily on those sources without exploring others. Eventually, this will backfire, especially as you reach more difficult courses. During my first three years I depended almost exclusively on lectures and problem sets, and so when I took a course during my fourth year with terrible lectures and very difficult problem sets, I struggled – because I couldn't get much out of the lectures, when I attempted the problems I rarely understood enough of the concepts to even begin. If I had understood at the time how to take

advantage of other resources, however, I would've been fine – we had an excellent TA who made himself available for one-on-one tutoring several hours each week, there were many great texts available on the subject, and there were several bright and friendly students in the class with whom I could've worked on problem sets or discussed the lectures, but because I did not take advantage of any of these other opportunities I had a much more difficult time than necessary with the course.

I should emphasize here the importance of working with other students. The further you go in mathematics, the more important this becomes, to the point where in graduate school many people would say that your fellow students are your single most important resource for learning mathematics. Working with other students does not mean copying answers, but rather discussing them. It is about sharing ideas. If you are having difficulties solving a problem, then someone else probably is too and perhaps together you can combine your approaches to reach a solution. Even if you can solve a problem, there are often many different ways to solve a single problem or even to look at the same solution, and by discussing the problem with other students you are exposed to many more of these and thus your overall understanding is greatly improved. Meeting regularly with students from your courses just to talk about the math you're doing and share perspectives is always a good idea.

0.4. **Don't ask for help before you've made an honest effort.** There is an infamous story at the U of A about a student who was ejected from the upper division tutoring room when the professor working that day asked him to define one of the terms in the statement of the problem that he had asked for help on and the student couldn't do it. Your sympathy should not be with the student in this case, although his mistake was a common one: he assumed that because he didn't understand how to do the problem immediately after reading it that there was no way he could solve it by himself. If you don't know where to begin on a problem, however, there are many things you should do before asking for help. First, go back and read the relevant section in the book, and really read it, don't just skim it. If there are examples in the book then try to work them out on your own, and if you can't, look at where you're getting stuck and if possible go back even further in the book to work on the concept you are getting stuck on. Go over your lecture notes. Try to rephrase the problem in a different way. If possible, discuss it with other students who are working on the same problem first before asking a professor. It is important to recognize when you need help and to be ready to ask for it, however, you're only hurting yourself if you don't make a real effort to think about the problem yourself first.

0.5. **Don't get stuck on one idea.** When working on a difficult problem, it's easy to get stuck on one idea. If this idea isn't the right one, you can waste a lot of time going around in the same circles. While its important to have perserverance, if you haven't made any progress on a problem in half an hour to an hour, then it's time to think about something else for a little while so that you can come back to it later with fresh ideas. This is particularly important later on for difficult problems in proof-based courses. In particular, you should

0.6. **Start early on homework sets.** This serves two purposes – if you get stuck on something, it gives you time to take a break and let your mind relax so that you can see new angles and other ways to approach a problem. Second, it gives

you time to find help from someone else if there's a problem that really stumps you even after you've made an honest effort.

0.7. **Learn things well the first time around.** Sometimes it's easy to get away with not learning something well, especially in your first and second year classes if you have a good memory and/or are quick. Remember that it is possible to learn something well without getting an A, and it is possible to get an A without learning something well (I have done both). The first case is not something you should worry about – good grades are important for many things, but if you make a habit of learning things well, then you will usually get good grades. The second case is something to watch out for, though. In particular, for foundational courses like calculus and linear algebra, it is important to really cultivate as strong of an understanding as you can – which often means, for instance, doing more difficult problems in your book which your professor did not assign (you can ask your professor for suggestions). As you continue on in mathematics, if you did not learn your early courses well then later on you will regret not having built a more solid foundation.